Received: 21 October 2021 | Revised: 24 August 2022 | Accepted: 15 December 2022

DOI: 10.1002/acs.3551

RESEARCH ARTICLE

WILEY

Event-triggered asynchronous filtering for networked fuzzy non-homogeneous Markov jump systems with dynamic quantization

Jin Zhu[1] | Zhi Xie[1] | Yun-Bo Zhao[1,2] | Geir E. Dullerud[3]

[1]Department of Automation, University of Science and Technology of China, Hefei, China

[2]Institute of Artificial Intelligence, Hefei Comprehensive National Science Center, Hefei, China

[3]Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, Champaign, Illinois, USA

Correspondence
Jin Zhu, Department of Automation, University of Science and Technology of China, Hefei 230027, China.
Email: jinzhu@ustc.edu.cn

Funding information
Anhui Provincial Natural Science Foundation, Grant/Award Number: 2008085MF198; Institute of Artificial Intelligence, Grant/Award Number: 21KT010; National Key Research and Development Project, Grant/Award Number: 2018AAA0100802

Summary

The asynchronous filtering problem for networked fuzzy non-homogeneous Markov jump systems is investigated, with the consideration of packet dropout and quantization. An event-triggered dynamic quantization scheme is proposed, and a stability criterion is given to ensure the stochastic stability of the filtering error systems with desired extended dissipative performance. This performance provides a unified framework in the sense that it can degenerate to H_∞, $l_2 - l_\infty$, dissipativity and passivity filter, respectively, under certain parameter sets. Furthermore, an asynchronous filter design method with extended dissipative performance is given based on the proposed stability criterion. In this method, the existence of the quantizer is ensured by dynamic quantization levels and the solution scope of the filter is enlarged by a free-connection weighting matrices method. The theoretical design and analysis is finally evaluated using a practical example.

KEYWORDS

asynchronous filtering, dynamic quantization, event-triggered, extended dissipative performance, free-connection weighting matrices, fuzzy non-homogeneous Markov jump systems

1 | INTRODUCTION

Markov jump systems (MJSs) have already proved itself useful in modeling many practical systems such as solar power receiver systems,[1] manufacturing systems[2] and circuit systems,[3] whose dynamics are typically subject to abrupt changes due to environmental disturbances, component failures, structural instability, and so forth.[1,4,5] In MJSs, these abrupt changes are modelled by a Markov chain governed switch rule among the “modes”, where the switch rule is usually described by a mode transition probability matrix (MTPM), and a mode is referring to a subsystem of the MJS whose dynamics are relatively stable. For homogeneous MJSs, that is, MJSs with time-invariant MTPM, existing achievements can refer to Costa et al.,[1] Mesquita,[6] Geromel et al.[7] and Zhu et al.[8] for the stability and stabilization, filtering, minimax control and mode feedback control of MJSs. On the other hand, non-homogeneous MJSs, that is, MJSs with time-varying MTPM, are more general and challenging. Several fundamental techniques include, the piecewise homogeneous approach which assumes the split of the MTPM into piecewise time-invariant ones,[2,9] the polytopic approach based on the polynomial cumulative form of the time-varying MTPM,[10-13] the dwell-time switching approach where the MTPM depends on the operation time,[14] and the norm-bounded uncertainties approach designed for the norm-bounded MTPM.[15,16] Based on these techniques, achievements have been made to address issues in non-homogeneous MJSs including stability analysis,[10,14] model reduction,[11] filter design,[12,13] and so forth. Especially, for the filtering problem, the mode-independent H_∞ filter design for nonlinear non-homogeneous MJSs with the multiplicative noises,[17] and the mode-dependent $l_2 - l_\infty$ filtering for uncertain non-homogeneous MJSs by using polytope Lyapunov function,[18] have recently been reported.

Networked MJSs have recently attracted much attention due to their simple installation and easy operation. But the advantages are not achieved at no cost: the unreliable transmissions and limited network bandwidth of the introduced communication networks mean that, information can be both lost due to packet dropout, and quantized due to the digital transmission. This brings to networked MJSs two challenges. Firstly, due to packet dropout, the modes available to the filter may not synchronize with original modes, which accounts for the asynchronization phenomenon in practice.[15] The event-triggered scheme has been applied in the filtering issue for homogeneous MJSs,[19,20] and for time-varying MTPM,[21] showing its prevalence in networked MJSs. Secondly, quantization inevitably degrades system performance, especially the so-call static quantization,[22] and hence dynamic quantization is attracting but there are few achievements for networked MJSs.[23] That is, the filter problem for networked MJSs with packet dropout and quantization error, remains far from solved till today.

In this paper, we endeavor to investigate the filtering problem of networked Fuzzy Non-homogeneous Markov Jump Systems(FN-MJSs) with both packet dropout and quantization. Firstly, we design an event-triggered scheme and a dynamic quantization method to reduce bandwidth occupation. Secondly, we propose an asynchronous extended dissipativity filter under imperfect premise matching. We want to mention that the proposed extended dissipativity filter is a generalization of existing results since the well-known H_∞, $l_2 - l_\infty$, dissipativity and passivity filters are its special cases. Furthermore, a sufficient criterion for stochastic stability of error systems is proposed in the form of coupled linear matrix inequalities. Based on the sufficient criterion, the asynchronous extended dissipativity filter is then designed using the relaxation method. This method, transforms the original problem with time-varying MTPM into the time-invariant case by introducing free-connection weighting matrices, and hence simplifies the design process of the filter via coupled linear matrix inequalities. Finally, practical examples are given to illustrate the effectiveness of the extended dissipativity filter, including both the H_∞ case and the $l_2 - l_\infty$ case. The main contributions of this paper are outlined as follows.

1. A more practical scenario is considered for the asynchronous filtering problem of networked non-homogeneous MJSs which suffer from unknown disturbance noise and packet dropout. In such a scenario, premise variables of FN-MJSs are not available to the filter due to packet dropouts. Existing results can be regarded as specific cases of the concerned model with some restrictions, for example, if the MTPM is limited to be time-invariant, it will degenerate into the homogeneous case;[22] when the packet-dropout problem is ignored, it turns to be the synchronous case.[24]
2. The extended dissipativity filter is investigated which provides a unified framework. By selecting appropriate parameters, the filtering design will degenerate to H_∞, $l_2 - l_\infty$, dissipativity and passivity filter respectively.[15,22,24] Moreover, when designing the extended dissipativity filter, conservatism has been reduced since no special requests are imposed on the form of time-varying MTPM.
3. It is the first attempt to consider both event-triggered scheme and dynamic quantization for MJSs. Compared with existing results whose quantization levels condition may be unsolvable,[23] the proposed quantizer design method in this paper can ensure its existence on condition that the stability criterion can be satisfied. Furthermore, with the introduction of the free-connection weighting matrices, the solution scope of asynchronous filtering can be enlarged.

The remainder of this paper is organized as follows: Section 2 describes the FN-MJSs model and the event-triggered asynchronous filter with dynamic quantization. In addition, some definitions of stability and extended dissipative performance about the FN-MJSs are also reviewed. Section 3 investigates the stabilization problem and the asynchronous filter design. Simulations are presented to verify the effectiveness of the proposed filter design in Section 4. Finally, a brief conclusion is drawn in Section 5.

Notation. The notations used in this paper are standard. $\mathbb{R}^n$ denotes n-dimensional Euclidean space, and $(\Omega, \mathscr{F}, \mathscr{P})$ is a probability space where Ω is the sample space, $\mathscr{F}$ is the σ-algebra of subsets of the sample space and $\mathscr{P}$ is the probability measure on $\mathscr{F}$. $\|\cdot\|$ denotes the Euclidean norm of a vector, or the induced Euclidean norm of a matrix. $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation. The superscript T represents the transposition of vector or matrix. The block diagonal matrix is denoted by $diag\{\cdot\}$. $X > 0 (< 0)$ indicates that the matrix is positive(negative) definite. $\mathbb{N}$ stands for the set of non-negative integers. The set of $n \times n$ (positive definite) symmetric matrices is denoted by $(\mathbb{S}_n^+)$ $\mathbb{S}_n$. In addition, in symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for the terms that introduced by symmetry. $\lfloor x \rfloor$ denotes the largest integer i such that $i \le x$. $X \otimes Y$ stands for Kronecker product of matrix X and Y. $\mathcal{H}e\{A\}$ is denoted by $A + A^{\mathrm{T}}$.

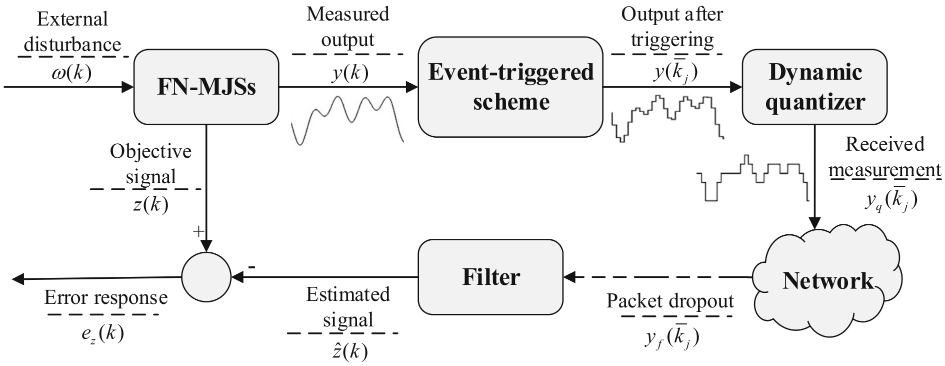

FIGURE 1 Filter framework of FN-MJSs

2 | PROBLEM FORMULATION AND PRELIMINARIES

The filter framework of FN-MJSs with disturbance noise is shown in Figure 1: For the concerned FN-MJSs, the output after triggering is firstly obtained with the event-triggered scheme applied. After triggering, the output remains a continuous value and is unable to be transmitted over network. For this reason, a dynamic quantizer is introduced. Taking into account the existence of packet dropout, the filter can only receive partial information. In general, we focus on the asynchronous filter design in this paper and investigate its extended dissipative performance.

2.1 | FN-MJSs model

Consider the following discrete-time FN-MJSs on the probability space $(\Omega, \mathscr{F}, \mathscr{P})$:

Plant Rule ζ: IF η_{1k} is $M_{\zeta 1}$, η_{2k} is $M_{\zeta 2}$, …, and η_{gk} is $M_{\zeta g}$, THEN

$$\begin{cases} x(k+1) = A_\zeta(r_k)x(k) + B_\zeta(r_k)\omega(k) \\ y(k) = C_\zeta(r_k)x(k) + D_\zeta(r_k)\omega(k) \\ z(k) = L_\zeta(r_k)x(k) + R_\zeta(r_k)\omega(k) \end{cases} \tag{1}$$

where $\eta_{ik}, i \in \{1, 2, \dots, g\}$ is the premise variable and $\eta(k) = (\eta_{1k}, \dots, \eta_{gk})$. And $M_{\zeta i}, \zeta \in \{1, 2, \dots, s\}$ is the fuzzy set in which s is the number of IF-THEN rules. $x(k) \in \mathbb{R}^{n_x}$ denotes the state vector, $\omega(k) \in \mathbb{R}^{n_\omega}$ denotes the disturbance noise belonging to $l_2[0, \infty)$, $y(k) \in \mathbb{R}^{n_y}$ denotes the measured output and $z(k) \in \mathbb{R}^{n_z}$ is the objective signal to be estimated. r_k is a non-homogeneous Markov chain defined on probability space $(\Omega, \mathscr{F}, \mathscr{P})$, taking values in $\mathscr{S}_1 = \{1, 2, \dots, s_1\}$ with time-varying MTPM $\Pi = [\pi_{ab}(k)]_{s_1 \times s_1}$ satisfying:

$$\pi_{ab}(k) = Pr(r_{k+1} = b | r_k = a);$$
$$\pi_{ab}(k) \geq 0, \quad \sum_{b=1}^{s_1} \pi_{ab}(k) = 1, \qquad a, b \in \mathscr{S}_1$$

where $\pi_{ab}(k)$ is the transition probability from mode a at time k to mode b at time $(k+1)$. And $A_\zeta(r_k), B_\zeta(r_k), C_\zeta(r_k)$, $D_\zeta(r_k), L_\zeta(r_k)$, and $R_\zeta(r_k)$ are known matrices with appropriate dimensions. For simplification, we write $A_\zeta(r_k), B_\zeta(r_k), C_\zeta(r_k), D_\zeta(r_k), L_\zeta(r_k), R_\zeta(r_k)$ as $A_{\zeta a}, B_{\zeta a}, C_{\zeta a}, D_{\zeta a}, L_{\zeta a}, R_{\zeta a}$ if $r_k = a, a \in \mathscr{S}_1$.

Considering the FN-MJSs (1), if $r_k = a, a \in \mathscr{S}_1$, we have the following systems in a compact form:

$$\begin{cases} x(k+1) = A_{ha}x(k) + B_{ha}\omega(k) \\ y(k) = C_{ha}x(k) + D_{ha}\omega(k) \\ z(k) = L_{ha}x(k) + R_{ha}\omega(k) \end{cases} \tag{2}$$

with

$$h_\zeta(\eta(k)) = \frac{\prod_{i=1}^{g} M_{\zeta i}(\eta_{ik})}{\sum_{\zeta=1}^{s}\prod_{i=1}^{g} M_{\zeta i}(\eta_{ik})}.$$

$$A_{ha} = \sum_{\zeta=1}^{s} h_\zeta(\eta(k))A_{\zeta a}, B_{ha} = \sum_{\zeta=1}^{s} h_\zeta(\eta(k))B_{\zeta a}, C_{ha} = \sum_{\zeta=1}^{s} h_\zeta(\eta(k))C_{\zeta a},$$

$$D_{ha} = \sum_{\zeta=1}^{s} h_\zeta(\eta(k))D_{\zeta a}, L_{ha} = \sum_{\zeta=1}^{s} h_\zeta(\eta(k))L_{\zeta a}, R_{ha} = \sum_{\zeta=1}^{s} h_\zeta(\eta(k))R_{\zeta a}$$

where $M_{\zeta i}(\eta_{ik})$ is the grade of membership of η_{ik} in $M_{\zeta i}$. $h_\zeta(\eta(k))$ is the membership function. It is easy to have that $\sum_{\zeta=1}^{s} h_\zeta(\eta(k)) = 1$ and for any $\zeta \in \{1, 2, \dots, s\}$, $h_\zeta(\eta(k)) \geq 0$.

2.2 | Event-triggered asynchronous filter with dynamic quantization

In this subsection, the event-triggered asynchronous filter with dynamic quantization is proposed. Firstly we give the form of the event-triggered scheme to decide instants $\bar{k}_{j+1}, j \in \mathbb{N}$, whose function is to transmit the measured output for filter updating,

$$\bar{k}_{j+1} = \min_{k \geq \bar{k}_j} \left\{ k | e_y^{\mathrm{T}}(k)\Phi_a e_y(k) \geq \sigma y^{\mathrm{T}}(\bar{k}_j)\Phi_a y(\bar{k}_j) \right\} \tag{3}$$

here the triggered error $e_y(k) = y(k) - y(\bar{k}_j)$, $\Phi_a > 0$ is a matrix to be determined and $\sigma > 0$ is a given constant. By introducing the event-triggered scheme, we collect data only at the sampling moment, which avoids data sampling effectively at every moment as in time-triggered scheme.

Secondly we consider the quantization problem. Differing from the existing efforts, we use the dynamic quantization instead of the static quantization to mitigate system performance degradation:

$$q_\mu(y(\bar{k}_j)) \triangleq \mu q\left(\frac{y(\bar{k}_j)}{\mu}\right) \tag{4}$$

According to the works of Liberzon et al.[25] we assume that there exist real numbers $\mathcal{M} > \Delta > 0$, such that the following two conditions hold:

$$\begin{aligned} &\textit{if } \|y(\bar{k}_j)\| \leq \mathcal{M}\mu, \quad \textit{then } \left\| \mu q\left(\frac{y(\bar{k}_j)}{\mu}\right) - y(\bar{k}_j) \right\| \leq \Delta\mu \\ &\textit{if } \|y(\bar{k}_j)\| > \mathcal{M}\mu, \quad \textit{then } \left\| \mu q\left(\frac{y(\bar{k}_j)}{\mu}\right) \right\| > (\mathcal{M} - \Delta)\mu \end{aligned} \tag{5}$$

where $\mathcal{M}$, μ and Δ are the dynamic quantization ranges, levels and error bounds, respectively.

According to the form of dynamic quantization (4), let $y_q(\bar{k}_j)$ be the signal after quantization. Then there is

$$y_q(\bar{k}_j) = q_\mu(y(\bar{k}_j))$$

where $y(\bar{k}_j)$ is the signal to be quantized at the moment $k = \bar{k}_j$. When $k = \bar{k}_j$, define the system's quantization error as:

$$e_\mu(k) = y_q(k) - y(k)$$

Finally, due to network-induced limitations, packet dropout happens unavoidably, which means $y_q(\bar{k}_j)$, the signal after quantization, is measured intermittently. This random phenomenon can be modeled as a Bernoulli process

$$y_f(\bar{k}_j) = \alpha_{\bar{k}_j} y_q(\bar{k}_j)$$

with probability distribution $Pr\{\alpha_{\bar{k}_j} = 1\} = \mathbb{E}\{\alpha_{\bar{k}_j}\} = \alpha(\alpha \in [0,1])$. If the number of IF-THEN rules of the FN-MJSs is available to the filter, based on the non-parallel distributed compensation,[26,27] we consider the following fuzzy full-order filter:

$$\begin{cases} \hat{x}(k+1) = A_{fh}(\rho_k)\hat{x}(k) + B_{fh}(\rho_k)y_f(\bar{k}_j) \\ \hat{z}(k) = C_{fh}(\rho_k)\hat{x}(k) \end{cases} \tag{6}$$

Here $\rho_k \in \mathcal{S}_2 = \{1, 2, \dots, s_2\}$ is a non-homogeneous discrete-time Markov chain which represents the stochastic switching of the filter mode, and its time-varying MTPM is described by $\Pi = [\varpi_{mn}^{r_{k+1}}(k)]$:

$$\varpi_{mn}^{r_{k+1}}(k) = Pr(\rho_{k+1} = n | \rho_k = m), \quad m, n \in \mathcal{S}_2$$

such that $\varpi_{mn}^{r_{k+1}}(k) \geq 0$ and $\sum_{n=1}^{s_2} \varpi_{mn}^{r_{k+1}}(k) = 1$. According to the approach proposed by Costa,[28] a detector can be designed to get the corresponding transition probabilities $\varpi_{mn}^{r_{k+1}}(k)$.

Generally speaking, the filter mode ρ_k is different from the original mode r_k due to the asynchronous phenomenon, and thus $\varpi_{mn}^{r_{k+1}}(k)$ is dependent on ρ_k and r_{k+1}. To characterize the relationship between ρ_k and r_k, the following conditional possibility is given:

$$Pr(r_{k+1} = b, \rho_{k+1} = n | r_k = a, \rho_k = m) = \varpi_{mn}^{b}(k)\pi_{ab}(k), \quad a, b \in \mathcal{S}_1, m, n \in \mathcal{S}_2$$

If $\rho_k = m$, similar to the original FN-MJSs (2), the asynchronous filter (6) can be represented as

$$\begin{cases} \hat{x}(k+1) = A_{fhm}\hat{x}(k) + B_{fhm}y_f(\bar{k}_j) \\ \hat{z}(k) = C_{fhm}\hat{x}(k) \end{cases} \tag{7}$$

with

$$g_j(\vartheta(k)) = \frac{\prod_{i=1}^{g} Q_{ji}(\vartheta_{ik})}{\sum_{j=1}^{s}\prod_{i=1}^{g} Q_{ji}(\vartheta_{ik})},$$
$$A_{fhm} = \sum_{j=1}^{s} g_j(\vartheta(k))A_{fjm}, B_{fhm} = \sum_{j=1}^{s} g_j(\vartheta(k))B_{fjm}, C_{fhm} = \sum_{j=1}^{s} g_j(\vartheta(k))C_{fjm}$$

where $Q_{ji}(\vartheta_{ik})$ is the grade of membership of ϑ_{ik} in Q_{ji}. $g_j(\vartheta(k))$ is the membership function satisfying $\vartheta(k) = (\vartheta_{1k}, \dots, \vartheta_{gk})$. It is clear that $\sum_{j=1}^{s} g_j(\vartheta(k)) = 1$ and $g_j(\vartheta(k)) \geq 0, \forall j \in \{1, 2, \dots, s\}$ which is different from the original system due to packet dropout.

Remark 1. The event-triggered asynchronous filter given in (7) is a general one which is also suitable for many existing scenarios. For example, by letting $\mathcal{S}_2$ contains only one element, that is, $s_2 = 1$, the concerned filter will degenerate into a mode-independent one where all original modes are unavailable.[17] Moreover, if $r_k = \rho_k$, then the concerned filter corresponds to the synchronous filter which means no packet dropout occurs.[24] Finally, for the specific case with $\rho_k = r_k$, that is, the original mode is available at time k but may be unavailable at time $k+1$, our model will degenerate to that of Tao et al.[22]

2.3 | Problem formulation

For the networked FN-MJSs (2) with event-triggered asynchronous filter (7), we formulate the problem as follows.

Let $e_x(k) = x(k) - \hat{x}(k)$, $e_z(k) = z(k) - \hat{z}(k)$, $\tilde{x}(k) = \begin{bmatrix} x^{\mathrm{T}}(k) & e_x^{\mathrm{T}}(k) \end{bmatrix}^{\mathrm{T}}$, then the error systems can be described as

$$\begin{cases} \tilde{x}(k+1) = \tilde{A}_{ha}\tilde{x}(k) + \tilde{B}_{ha}\omega(k) + \tilde{E}_{ha}(e_\mu(\bar{k}_j) - e_y(k)) + \epsilon(k) \\ e_z(k) = \tilde{C}_{ha}\tilde{x}(k) + \tilde{D}_{ha}\omega(k) \end{cases} \tag{8}$$

where

$$\tilde{A}_{ha}=\sum_{\zeta=1}^{s}\sum_{j=1}^{s}h_{\zeta}(\eta(k))g_j(\vartheta(k))\tilde{A}_{\zeta j},\tilde{B}_{ha}=\sum_{\zeta=1}^{s}\sum_{j=1}^{s}h_{\zeta}(\eta(k))g_j(\vartheta(k))\tilde{B}_{\zeta j},\tilde{C}_{ha}=\sum_{\zeta=1}^{s}\sum_{j=1}^{s}h_{\zeta}(\eta(k))g_j(\vartheta(k))\tilde{C}_{\zeta j},$$

$$\tilde{D}_{ha}=\sum_{\zeta=1}^{s}\sum_{j=1}^{s}h_{\zeta}(\eta(k))g_j(\vartheta(k))\tilde{D}_{\zeta j},\tilde{E}_{ha}=\sum_{j=1}^{s}g_j(\vartheta(k))\tilde{E}_j,$$

$$\tilde{A}_{\zeta j}=\begin{bmatrix}A_{\zeta a} & 0\\ A_{\zeta a}-\alpha B_{fjm}C_{\zeta a}-A_{fjm} & A_{fjm}\end{bmatrix},\tilde{B}_{\zeta j}=\begin{bmatrix}B_{\zeta a}\\ B_{\zeta a}-\alpha B_{fjm}D_{\zeta a}\end{bmatrix},\tilde{E}_j=\begin{bmatrix}0\\ -\alpha_{\bar{k}_j}B_{fjm}\end{bmatrix},$$

$$\tilde{C}_{\zeta j}=\begin{bmatrix}L_{\zeta a}-C_{fjm} & C_{fjm}\end{bmatrix},\tilde{D}_{\zeta j}=R_{\zeta a},\epsilon(k)=(\alpha-\alpha_{\bar{k}_j})(B_{fhm}C_{ha}x(k)+B_{fhm}D_{ha}\omega(k)).$$

Noticing the mathematical expectation of the term $\epsilon(k)$ is zero, that is, $\mathbb{E}\{\epsilon(k)\}\equiv 0$, and taking mathematical expectation of $\tilde{E}_{ha}$, we have

$$\bar{E}_{ha}\triangleq\mathbb{E}\left\{\tilde{E}_{ha}\right\}=\sum_{j=1}^{s}g_j(\vartheta(k))\bar{E}_j,\bar{E}_j=\begin{bmatrix}0\\ -\alpha B_{fjm}\end{bmatrix}$$

Noticing that both the filter modes and the original modes are governed by the non-homogeneous Markov chain, the time-varying MTPMs can then be described as follows.

Denote the finite set $\mathcal{S}_1=\mathcal{S}_{\mathcal{IV}}^{b}\cup\mathcal{S}_{\mathcal{V}}^{b}$, $\mathcal{S}_2=\mathcal{S}_{\mathcal{IV}}^{n}\cup\mathcal{S}_{\mathcal{V}}^{n}$, $\forall b\in\mathcal{S}_1, n\in\mathcal{S}_2$ with

$$\begin{aligned}&\mathcal{S}_{\mathcal{IV}}^{b}\triangleq\{b|\pi_{ab}(k)=\pi_{ab}\ is\ time-invariant\},\mathcal{S}_{\mathcal{V}}^{b}\triangleq\{b|\pi_{ab}(k)\in[\check{\pi}_{ab},\hat{\pi}_{ab}]\ is\ time-varying\}\\ &\mathcal{S}_{\mathcal{IV}}^{n}\triangleq\{n|\varpi_{mn}^{b}(k)=\varpi_{mn}^{b}\ is\ time-invariant\},\mathcal{S}_{\mathcal{V}}^{n}\triangleq\{n|\varpi_{mn}^{b}(k)\in[\check{\varpi}_{mn}^{b},\hat{\varpi}_{mn}^{b}]\ is\ time-varying\}\end{aligned}\tag{9}$$

Here $[\check{\pi}_{ab},\hat{\pi}_{ab}]$, $[\check{\varpi}_{mn}^{b},\hat{\varpi}_{mn}^{b}]$ are the known bounds of the time-varying terms of the MTPMs.

For notational convenience, we represent

$$\begin{aligned}[X(a,b)]_{b\in\{b_1,b_2,\ldots,b_n\}}&=\begin{bmatrix}X^{\mathrm{T}}(a,b_1) & X^{\mathrm{T}}(a,b_2) & \ldots & X^{\mathrm{T}}(a,b_n)\end{bmatrix}^{\mathrm{T}},\\ [X(a,b)]_{b\in\{b_1,b_2,\ldots,b_n\}}^{D}&=diag\{X(a,b_1),X(a,b_2),\ldots,X(a,b_n)\},\\ \sum_{n\in\mathcal{S}_{IV}^{n}}\varpi_{mn}^{b}&=\Psi_{IV},\sum_{b\in\mathcal{S}_{IV}^{b}}\pi_{ab}=\Pi_{IV},[\varpi_{mn}^{b}(k)]_{n\in\mathcal{S}_{V}^{n}}=\Psi_{V},\\ [\pi_{ab}(k)]_{b\in\mathcal{S}_{V}^{b}}&=\Pi_{V},\check{\varpi}_{mn}^{b}+\hat{\varpi}_{mn}^{b}=\overline{\varpi}_{mn}^{b},\check{\pi}_{ab}+\hat{\pi}_{ab}=\overline{\pi}_{ab}.\end{aligned}\tag{10}$$

Before the designing of the asynchronous filter, the following two definitions are given:

Definition 1 (stochastic stability; Reference 12). Taking into account the noise-free form of the error systems (8) with the same coefficient matrices which is described as follows:

$$\begin{cases}\tilde{x}(k+1)=\tilde{A}_{ha}\tilde{x}(k)+\tilde{E}_{ha}(e_{\mu}(\bar{k}_j)-e_y(k))+\epsilon(k)\\ e_z(k)=\tilde{C}_{ha}\tilde{x}(k)\end{cases}\tag{11}$$

the noise-free error systems (11) are said to be stochastically stable with respect to any initial state $(\tilde{x}(0),r_0,\rho_0)$, if the following inequality holds:

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}\|\tilde{x}(k)\|^2\Big|\tilde{x}(0),r_0,\rho_0\right\}<\infty$$

Definition 2 (extended dissipative performance; Reference 29). For known real matrices $\mathcal{U}_1=-(\mathcal{U}_1^{+})^{\mathrm{T}}\mathcal{U}_1^{+}\leq 0$, $\mathcal{U}_2$, $\mathcal{U}_3=\mathcal{U}_3^{\mathrm{T}}$ and $\mathcal{U}_4=(\mathcal{U}_4^{+})^{\mathrm{T}}\mathcal{U}_4^{+}\geq 0$ satisfying

$$\begin{aligned}\|\tilde{D}_{ha}\|\|\mathcal{U}_4\|&=0,\\ (\|\mathcal{U}_1\|+\|\mathcal{U}_2\|)\|\mathcal{U}_4\|&=0,\\ \tilde{D}_{ha}^{\mathrm{T}}\mathcal{U}_1\tilde{D}_{ha}+\mathcal{H}e\left\{\mathcal{U}_2^{\mathrm{T}}\tilde{D}_{ha}\right\}+\mathcal{U}_3&>0,\end{aligned}\tag{12}$$

the error systems (8) are said to be extended dissipative if the following inequality holds for any integer τ and $k \in \{0, 1, \ldots, \tau\}$ under the zero initial condition:

$$\mathbb{E}\left\{\sum_{k=0}^{\tau} J(k)\right\} = \mathbb{E}\left\{\sum_{k=0}^{\tau}\left(e_z^{\mathrm{T}}(k)\mathcal{U}_1 e_z(k) + 2e_z^{\mathrm{T}}(k)\mathcal{U}_2\omega(k) + \omega^{\mathrm{T}}(k)\mathcal{U}_3\omega(k)\right)\right\} > \sup_{0\le k\le\tau}\mathbb{E}\{e_z^{\mathrm{T}}(k)\mathcal{U}_4 e_z(k)\}$$

In general, the problem investigated in this paper is given as follows:

Problem 1. Considering the networked FN-MJSs (2) with dynamic quantization and packet dropout, the goal is to find the event-triggered asynchronous filter (7) with coefficient matrices A_{fjm}, B_{fjm}, C_{fjm} as well as appropriate quantization levels condition $\mu(k)$ such that the error systems (8) are stochastically stable with a desired extended dissipative performance.

Namely, for the noise-free error systems (11), the following inequality holds:

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}\|\tilde{x}(k)\|^2 \middle| \tilde{x}(0), r_0, \rho_0\right\} < \infty$$

and for the error systems (8), the following inequality holds:

$$\mathbb{E}\left\{\sum_{k=0}^{\tau}\left(e_z^{\mathrm{T}}(k)\mathcal{U}_1 e_z(k) + 2e_z^{\mathrm{T}}(k)\mathcal{U}_2\omega(k) + \omega^{\mathrm{T}}(k)\mathcal{U}_3\omega(k)\right)\right\} > \sup_{0\le k\le\tau}\mathbb{E}\left\{e_z^{\mathrm{T}}(k)\mathcal{U}_4 e_z(k)\right\}$$

3 | MAIN RESULTS

Taking into account Problem 1, two essential challenges are to be solved: (1) the quantization levels condition and the sufficient conditions of stochastic stability, and (2) the asynchronous filter design with extended dissipative performance.

3.1 | Stability criterion

We firstly focus on the stochastic stability for the noise-free error systems (11) with the event-triggered scheme and the dynamic quantization technology.

3.1.1 | Stochastic stability

A sufficient criterion of the stochastic stability is addressed by the following Lemma 1 together with the quantization levels condition given:

Lemma 1. *For given scalars quantization range $\mathcal{M}$, error bound Δ, a positive number $\delta > 0$ and a constant $\sigma > 0$, the noise-free error systems (11) are stochastically stable if there exist small enough scalar $\beta > 0$ and matrices $P(a, m) > 0, \Phi_a > 0$ such that the following condition holds*:

$$\overline{\Xi} < 0 \tag{13}$$

where

$$\overline{\Xi} = \begin{bmatrix} \overline{\Xi}_1 & \overline{\Xi}_2 \\ * & \overline{\Xi}_3 \end{bmatrix}, \overline{\Xi}_1 = \begin{bmatrix} -P(a,m) + \sigma\vec{C}_{ha}^{\mathrm{T}}\Phi_a\vec{C}_{ha} & -\sigma\vec{C}_{ha}^{\mathrm{T}}\Phi_a & 0 \\ * & (\sigma-1)\Phi_a + (\delta+1)^2 I & 0 \\ * & * & -\frac{\mathcal{M}^2}{\Delta^2}I \end{bmatrix} + \beta I,$$

$$\overline{\Xi}_2 = \begin{bmatrix} \tilde{A}_{ha}^{\mathrm{T}}\tilde{P}(a,m) & (\delta+1)\vec{C}_{ha}^{\mathrm{T}} \\ -\bar{E}_{ha}^{\mathrm{T}}\tilde{P}(a,m) & 0 \\ \bar{E}_{ha}^{\mathrm{T}}\tilde{P}(a,m) & 0 \end{bmatrix}, \overline{\Xi}_3 = diag\{-\tilde{P}(a,m), -I\}$$

with $\vec{C}_{ha} = [C_{ha} \quad 0]$, $\tilde{P}(a,m) = \sum_{b=1}^{s_1}\sum_{n=1}^{s_2}\pi_{ab}(k)\varpi_{mn}^{b}(k)P(b,n)$. *The quantization levels condition for parameters* $\mu(k)$ *are given as*

$$\frac{1}{\mathcal{M}}\|y(\bar{k}_j)\| \leq \mu(k) \leq \frac{\delta+1}{\mathcal{M}}\|y(\bar{k}_j)\|, \quad \delta > 0 \tag{14}$$

Proof. We construct the following Lyapunov function:[30]

$$V(\tilde{x}(k), r_k = a, \rho_k = m) = \tilde{x}^{\mathrm{T}}(k)P(a,m)\tilde{x}(k) \tag{15}$$

For simplification, we write $V(\tilde{x}(k), r_k = a, \rho_k = m)$ as $V(k)$. Then, the forward difference operator of $V(k)$ can be presented as

$$\begin{aligned}\Delta V(k) &= V(\tilde{x}(k+1), b, n|a,m) - V(k)\\ &= \tilde{x}^{\mathrm{T}}(k+1)\tilde{P}(a,m)\tilde{x}(k+1) - \tilde{x}^{\mathrm{T}}(k)P(a,m)\tilde{x}(k)\end{aligned}$$

where

$$\tilde{P}(a,m) = \sum_{b=1}^{s_1}\sum_{n=1}^{s_2}\pi_{ab}(k)\varpi_{mn}^{b}(k)P(b,n),$$

Taking the mathematical expectation of $\Delta V(k)$, for the noise-free error systems (11), there is

$$\begin{aligned}\mathbb{E}\{\Delta V(k)\} &= \mathbb{E}\{\tilde{x}^{\mathrm{T}}(k)\left(\tilde{A}_{ha}^{\mathrm{T}}\tilde{P}(a,m)\tilde{A}_{ha} - P(a,m)\right)\tilde{x}(k) + \left(e_{\mu}^{\mathrm{T}}(\bar{k}_j) - e_{y}^{\mathrm{T}}(k)\right)\bar{E}_{ha}^{\mathrm{T}}\tilde{P}(a,m)\bar{E}_{ha}\left(e_{\mu}(\bar{k}_j) - e_{y}(k)\right)\\ &\quad + 2\tilde{x}^{\mathrm{T}}(k)\left(\tilde{A}_{ha}^{\mathrm{T}}\tilde{P}(a,m)\bar{E}_{ha}\right)\left(e_{\mu}(\bar{k}_j) - e_{y}(k)\right)\}\\ &= \mathbb{E}\left\{\xi^{\mathrm{T}}(k)\Xi_0\xi(k)\right\}\end{aligned} \tag{16}$$

where $\xi(k) = \begin{bmatrix}\tilde{x}^{\mathrm{T}}(k) & e_{y}^{\mathrm{T}}(k) & e_{\mu}^{\mathrm{T}}(\bar{k}_j)\end{bmatrix}^{\mathrm{T}}$. According to the event-triggered condition (3), for any $k \in [k_i, k_{i+1}]$, we have:

$$\begin{aligned}\varphi &\triangleq \mathbb{E}\left\{\sigma y^{\mathrm{T}}(k)\Phi_a y(k) - 2\sigma y^{\mathrm{T}}(k)\Phi_a e_y(k) + (\sigma-1)e_{y}^{\mathrm{T}}(k)\Phi_a e_y(k)\right\}\\ &= \mathbb{E}\left\{\xi^{\mathrm{T}}(k)\left\{\sigma\begin{bmatrix}\vec{C}_{ha} & 0 & 0\end{bmatrix}^{\mathrm{T}}\Phi_a\begin{bmatrix}\vec{C}_{ha} & 0 & 0\end{bmatrix} - 2\sigma\begin{bmatrix}\vec{C}_{ha} & 0 & 0\end{bmatrix}^{\mathrm{T}}\Phi_a\begin{bmatrix}0 & I & 0\end{bmatrix} + (\sigma-1)\begin{bmatrix}0 & I & 0\end{bmatrix}^{\mathrm{T}}\Phi_a\begin{bmatrix}0 & I & 0\end{bmatrix}\right\}\xi(k)\right\}\\ &= \mathbb{E}\left\{\xi^{\mathrm{T}}(k)\Xi_1\xi(k)\right\} \geq 0\end{aligned} \tag{17}$$

Based on the condition (5), if $\|y(\bar{k}_j)\| \leq \mathcal{M}\mu(k)$, the error $e_{\mu}(\bar{k}_j)$ caused by quantization is bounded. Otherwise, $e_{\mu}(\bar{k}_j)$ is unbounded since $\|y(\bar{k}_j)\| > \mathcal{M}\mu(k)$. In general, $e_{\mu}(\bar{k}_j)$ can be presented as follows,

$$\|e_{\mu}(\bar{k}_j)\| \leq \mu_{max}\Delta = \frac{(\delta+1)\Delta}{\mathcal{M}}\|y(\bar{k}_j)\| = \frac{(\delta+1)\Delta}{\mathcal{M}}\|y(k) - e_y(k)\| \leq \frac{(\delta+1)\Delta}{\mathcal{M}}(\|e_y(k)\| + \|y(k)\|), \quad \delta > 0 \tag{18}$$

here $e_y(k)$ is the triggered error as defined in (3). Combining (2) with (18), one has

$$\xi^{\mathrm{T}}(k)\left\{\begin{bmatrix}\vec{C}_{ha} & 0 & 0\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\vec{C}_{ha} & 0 & 0\end{bmatrix} + diag\{0, I, 0\} - diag\left\{0, 0, \frac{\mathcal{M}^2}{(\delta+1)^2\Delta^2}I\right\}\right\}\xi(k) = \xi^{\mathrm{T}}(k)\Xi_2\xi(k) \geq 0 \tag{19}$$

According to the matrix inequality (13) and applying Schur Complement, we can obtain

$$\Xi_0 + \Xi_1 + (\delta+1)^2\Xi_2 < -\beta I \tag{20}$$

Based on the definition of negative-definite matrix,[31] (20) implies

$$\mathbb{E}\left\{\xi^{\mathrm{T}}(k)\left(\Xi_0 + \Xi_1 + (\delta+1)^2\Xi_2\right)\xi(k)\right\} = \mathbb{E}\left\{\xi^{\mathrm{T}}(k)\Xi_0\xi(k)\right\} + \mathbb{E}\left\{\xi^{\mathrm{T}}(k)\Xi_1\xi(k)\right\} + (\delta+1)^2\mathbb{E}\left\{\xi^{\mathrm{T}}(k)\Xi_2\xi(k)\right\} < -\beta I$$

Based on (17) and (19) where $\delta > 0$, together with (16), we get

$$\mathbb{E}\left\{\xi^{\mathrm{T}}(k)\Xi_0\xi(k)\right\} = \mathbb{E}\left\{\Delta V(k)\right\} < -\beta I$$

That is

$$\mathbb{E}\left\{\Delta V(k)\right\} \leq -\beta\mathbb{E}\left\{\|\xi(k)\|^2\right\} \tag{21}$$

Summing up both sides of (21) from 0 to ∞,

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}\|\xi(k)\|^2\Big|\xi(0), r_0, \rho_0\right\} \leq \frac{1}{\beta}\mathbb{E}\left\{V(0)\right\} < \infty \tag{22}$$

Noticing that $\xi(k) = \begin{bmatrix}\tilde{x}^{\mathrm{T}}(k) & e_y^{\mathrm{T}}(k) & e_\mu^{\mathrm{T}}(\overline{k}_j)\end{bmatrix}^{\mathrm{T}}$, there is

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}\|\xi(k)\|^2\Big|\xi(0), r_0, \rho_0\right\} \geq \mathbb{E}\left\{\sum_{k=0}^{\infty}\|\tilde{x}(k)\|^2\Big|\tilde{x}(0), r_0, \rho_0\right\} \tag{23}$$

Combining (23) with (22), one has

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}\|\tilde{x}(k)\|^2\Big|\tilde{x}(0), r_0, \rho_0\right\} < \infty$$

According to Definition 1, the noise-free error systems (11) are stochastically stable. ■

3.1.2 | Extended dissipativity

In this part, we consider the error systems (8) with disturbance noise and focus on the extended dissipative performance. A sufficient criterion is addressed by the following theorem:

Theorem 1. *For given matrices $\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \mathcal{U}_4$ satisfying Definition 2, scalars quantization range $\mathcal{M}$, error bound Δ, a positive number $\delta > 0$ and a constant $\sigma > 0$, the error systems (8) are extended dissipative if there exist small enough scalar $\beta > 0$ and matrices $P(a, m) > 0$, $\Phi_a > 0$ such that the following conditions hold*:

$$\hat{\Xi} < 0 \tag{24}$$

$$\Lambda < 0 \tag{25}$$

where

$$\hat{\Xi} = \begin{bmatrix}\hat{\Xi}_1 & \hat{\Xi}_2\\ * & \hat{\Xi}_3\end{bmatrix}, \Lambda = \begin{bmatrix}-P(a, m) & \tilde{C}_{ha}^{\mathrm{T}}(\mathcal{U}_4^{+})^{\mathrm{T}}\\ * & -I\end{bmatrix},$$

$$\hat{\Xi}_1 = \begin{bmatrix}-P(a, m) + \sigma\vec{C}_{ha}^{\mathrm{T}}\Phi_a\vec{C}_{ha} & -\sigma\vec{C}_{ha}^{\mathrm{T}}\Phi_a & 0 & -\tilde{C}_{ha}^{\mathrm{T}}\mathcal{U}_2\\ * & (\sigma-1)\Phi_a + (\delta+1)^2 I & 0 & -\sigma D_{ha}^{\mathrm{T}}\Phi_a\\ * & * & -\frac{\mathcal{M}^2}{\Delta^2}I & 0\\ * & * & * & -\mathcal{H}e\{\tilde{D}_{ha}^{\mathrm{T}}\mathcal{U}_2\} - \mathcal{U}_3 + \sigma D_{ha}^{\mathrm{T}}\Phi_a D_{ha}\end{bmatrix} + \beta I,$$

$$\hat{\Xi}_2 = \begin{bmatrix}\tilde{A}_{ha}^{\mathrm{T}}\tilde{P}(a, m) & (\delta+1)\vec{C}_{ha}^{\mathrm{T}} & \tilde{C}_{ha}^{\mathrm{T}}(\mathcal{U}_1^{+})^{\mathrm{T}}\\ -\bar{E}_{ha}^{\mathrm{T}}\tilde{P}(a, m) & 0 & 0\\ \bar{E}_{ha}^{\mathrm{T}}\tilde{P}(a, m) & 0 & 0\\ \tilde{B}_{ha}^{\mathrm{T}}\tilde{P}(a, m) & (\delta+1)D_{ha}^{\mathrm{T}} & \tilde{D}_{ha}^{\mathrm{T}}(\mathcal{U}_1^{+})^{\mathrm{T}}\end{bmatrix}, \hat{\Xi}_3 = diag\{-\tilde{P}(a, m), -I, -I\}$$

with $\vec{C}_{ha} = [C_{ha} \quad 0]$, $\tilde{P}(a, m) = \sum_{b=1}^{s_1}\sum_{n=1}^{s_2}\pi_{ab}(k)\varpi_{mn}^{b}(k)P(b, n)$ *and the quantization levels condition given in (14).*

Proof. The Lyapunov function is defined in (15). Firstly we define the vector $\bar{\xi}^{\mathrm{T}}(k) = \begin{bmatrix}\tilde{x}^{\mathrm{T}}(k) & e_y^{\mathrm{T}}(k) & e_\mu^{\mathrm{T}}(\bar{k}_j) & \omega^{\mathrm{T}}(k)\end{bmatrix}$. Based on the Definition 2, there is

$$
\begin{aligned}
&\mathbb{E}\{\Delta V(k) - e_z^{\mathrm{T}}(k)\mathcal{U}_1 e_z(k) - 2e_z^{\mathrm{T}}(k)\mathcal{U}_2\omega(k) - \omega^{\mathrm{T}}(k)\mathcal{U}_3\omega(k)\}\\
&\quad = \mathbb{E}\{\bar{\xi}^{\mathrm{T}}(k)\{\begin{bmatrix}\tilde{A}_{ha} & -\bar{E}_{ha} & \bar{E}_{ha} & \tilde{B}_{ha}\end{bmatrix}^{\mathrm{T}}\tilde{P}(a,m)\begin{bmatrix}\tilde{A}_{ha} & -\bar{E}_{ha} & \bar{E}_{ha} & \tilde{B}_{ha}\end{bmatrix} - \begin{bmatrix}I & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}P(a,m)\begin{bmatrix}I & 0 & 0 & 0\end{bmatrix}\\
&\qquad - \begin{bmatrix}\tilde{C}_{ha} & 0 & 0 & \tilde{D}_{ha}\end{bmatrix}^{\mathrm{T}}\mathcal{U}_1\begin{bmatrix}\tilde{C}_{ha} & 0 & 0 & \tilde{D}_{ha}\end{bmatrix} - 2\begin{bmatrix}\tilde{C}_{ha} & 0 & 0 & \tilde{D}_{ha}\end{bmatrix}^{\mathrm{T}}\mathcal{U}_2\begin{bmatrix}0 & 0 & 0 & I\end{bmatrix}\\
&\qquad - \begin{bmatrix}0 & 0 & 0 & I\end{bmatrix}^{\mathrm{T}}\mathcal{U}_3\begin{bmatrix}0 & 0 & 0 & I\end{bmatrix}\}\bar{\xi}(k)\}\\
&\quad = \mathbb{E}\left\{\bar{\xi}^{\mathrm{T}}(k)\Xi_0\bar{\xi}(k)\right\}
\end{aligned}
\tag{26}
$$

The event-triggered condition for the error systems (8) with noise is different from the noise-free error systems (11), therefore, we have

$$
\begin{aligned}
\varphi &\triangleq \mathbb{E}\left\{\sigma y^{\mathrm{T}}(k)\Phi_a y(k) - 2\sigma y^{\mathrm{T}}(k)\Phi_a e_y(k) + (\sigma - 1)e_y^{\mathrm{T}}(k)\Phi_a e_y(k)\right\}\\
&= \mathbb{E}\{\bar{\xi}^{\mathrm{T}}(k)\{\sigma\begin{bmatrix}\vec{C}_{ha} & 0 & 0 & D_{ha}\end{bmatrix}^{\mathrm{T}}\Phi_a\begin{bmatrix}\vec{C}_{ha} & 0 & 0 & D_{ha}\end{bmatrix} - 2\sigma\begin{bmatrix}\vec{C}_{ha} & 0 & 0 & D_{ha}\end{bmatrix}^{\mathrm{T}}\Phi_a\begin{bmatrix}0 & I & 0 & 0\end{bmatrix}\\
&\quad + (\sigma - 1)\begin{bmatrix}0 & I & 0 & 0\end{bmatrix}^{\mathrm{T}}\Phi_a\begin{bmatrix}0 & I & 0 & 0\end{bmatrix}\}\bar{\xi}(k)\}\\
&= \mathbb{E}\left\{\bar{\xi}^{\mathrm{T}}(k)\Xi_1\bar{\xi}(k)\right\} \geq 0
\end{aligned}
\tag{27}
$$

For the error systems (8), according to (18), the inequality (19) can be rewritten as

$$
\bar{\xi}^{\mathrm{T}}(k)\left\{\begin{bmatrix}\vec{C}_{ha} & 0 & 0 & D_{ha}\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\vec{C}_{ha} & 0 & 0 & D_{ha}\end{bmatrix} + diag\{0, I, 0, 0\} - diag\left\{0, 0, \frac{\mathcal{M}^2}{(\delta+1)^2\Delta^2}I, 0\right\}\right\}\bar{\xi}(k) = \bar{\xi}^{\mathrm{T}}(k)\Xi_2\bar{\xi}(k) \geq 0
\tag{28}
$$

According to the condition (24), and using the Schur Complement, one has

$$
\Xi_0 + \Xi_1 + (\delta + 1)^2\Xi_2 < -\beta I \tag{29}
$$

Considering (26)–(29), similarly, $\mathbb{E}\{\bar{\xi}^{\mathrm{T}}(k)\Xi_0\bar{\xi}(k)\} < -\beta I$ is obtained. Recalling (26), we get

$$
\mathbb{E}\left\{e_z^{\mathrm{T}}(k)\mathcal{U}_1 e_z(k) + 2e_z^{\mathrm{T}}(k)\mathcal{U}_2\omega(k) + \omega^{\mathrm{T}}(k)\mathcal{U}_3\omega(k)\right\} > \mathbb{E}\left\{\Delta V(k)\right\} \tag{30}
$$

Under zero initial condition $V(0) = 0$, we sum up both sides of inequality (30) from 0 to τ. Considering $P(a, m) > 0$, we have

$$
\mathbb{E}\left\{\sum_{k=0}^{\tau}\left(e_z^{\mathrm{T}}(k)\mathcal{U}_1 e_z(k) + 2e_z^{\mathrm{T}}(k)\mathcal{U}_2\omega(k) + \omega^{\mathrm{T}}(k)\mathcal{U}_3\omega(k)\right)\right\} > \mathbb{E}\left\{V(\tau+1)\right\} \geq 0 \tag{31}
$$

1) If $\mathcal{U}_4$ is a zero matrix, for any integer k satisfying $0 \leq k \leq \tau$, we have

$$
\mathbb{E}\left\{V(\tau+1)\right\} \geq \mathbb{E}\left\{e_z^{\mathrm{T}}(k)\mathcal{U}_4 e_z(k)\right\} = 0
$$

Together with (31), we have

$$
\mathbb{E}\left\{\sum_{k=0}^{\tau}J(k)\right\} = \mathbb{E}\left\{\sum_{k=0}^{\tau}\left(e_z^{\mathrm{T}}(k)\mathcal{U}_1 e_z(k) + 2e_z^{\mathrm{T}}(k)\mathcal{U}_2\omega(k) + \omega^{\mathrm{T}}(k)\mathcal{U}_3\omega(k)\right)\right\} > \sup_{0\leq k\leq\tau}\mathbb{E}\left\{e_z^{\mathrm{T}}(k)\mathcal{U}_4 e_z(k)\right\}
$$

2) If $\mathcal{U}_4$ is not a zero matrix, there always exists an integer k_m, $(0 \leq k_m \leq \tau)$ such that the following condition is satisfied

$$\mathbb{E}\left\{e_z^{\mathrm{T}}(k_m)\mathcal{U}_4 e_z(k_m)\right\} = \sup_{0\leq k\leq \tau} \mathbb{E}\left\{e_z^{\mathrm{T}}(k)\mathcal{U}_4 e_z(k)\right\}$$

By Definition 2, we yield that $\tilde{D}_{ha}$, $\mathcal{U}_1$ and $\mathcal{U}_2$ are zero matrix. According to (12), $\mathcal{U}_3 > 0$. Then we obtain $e_z(k) = \tilde{C}_{ha}\tilde{x}(k)$. Furthermore, based on the condition (25), we have

$$\tilde{C}_{ha}^{\mathrm{T}}\mathcal{U}_4\tilde{C}_{ha} - P(a, m) < 0.$$

It derives that the following inequality holds

$$\mathbb{E}\left\{V(k_m)\right\} = \mathbb{E}\left\{\tilde{x}^{\mathrm{T}}(k_m)P(a, m)\tilde{x}(k_m)\right\} > \mathbb{E}\left\{\tilde{x}^{\mathrm{T}}(k_m)\tilde{C}_{ha}^{\mathrm{T}}\mathcal{U}_4\tilde{C}_{ha}\tilde{x}^{\mathrm{T}}(k_m)\right\} = \mathbb{E}\left\{e_z^{\mathrm{T}}(k_m)\mathcal{U}_4 e_z(k_m)\right\} \tag{32}$$

When $k_m = 0$, under zero initial condition, (32) yields

$$\mathbb{E}\left\{e_z^{\mathrm{T}}(k_m)\mathcal{U}_4 e_z(k_m)\right\} < \mathbb{E}\left\{V(0)\right\} = 0.$$

By (31), we have

$$\mathbb{E}\left\{\sum_{k=0}^{\tau}\left(e_z^{\mathrm{T}}(k)\mathcal{U}_1 e_z(k) + 2e_z^{\mathrm{T}}(k)\mathcal{U}_2\omega(k) + \omega^{\mathrm{T}}(k)\mathcal{U}_3\omega(k)\right)\right\} > \sup_{0\leq k\leq \tau}\mathbb{E}\left\{e_z^{\mathrm{T}}(k)\mathcal{U}_4 e_z(k)\right\}$$

When $0 < k_m \leq \tau$, $\mathcal{U}_3 > 0$, $\mathcal{U}_1$ and $\mathcal{U}_2$ are zero matrix, one has

$$\mathbb{E}\left\{\sum_{k=0}^{\tau}\left(e_z^{\mathrm{T}}(k)\mathcal{U}_1 e_z(k) + 2e_z^{\mathrm{T}}(k)\mathcal{U}_2\omega(k) + \omega^{\mathrm{T}}(k)\mathcal{U}_3\omega(k)\right)\right\} > \mathbb{E}\left\{\sum_{k=0}^{k_m-1}\left(e_z^{\mathrm{T}}(k)\mathcal{U}_1 e_z(k) + 2e_z^{\mathrm{T}}(k)\mathcal{U}_2\omega(k) + \omega^{\mathrm{T}}(k)\mathcal{U}_3\omega(k)\right)\right\}$$

Considering (30), we sum up both sides of inequality (30) from 0 to $k_m - 1$ which yields

$$\mathbb{E}\left\{\sum_{k=0}^{k_m-1}\left(e_z^{\mathrm{T}}(k)\mathcal{U}_1 e_z(k) + 2e_z^{\mathrm{T}}(k)\mathcal{U}_2\omega(k) + \omega^{\mathrm{T}}(k)\mathcal{U}_3\omega(k)\right)\right\} > \mathbb{E}\left\{V(k_m)\right\}$$

According to the above two inequalities, one has

$$\mathbb{E}\left\{\sum_{k=0}^{\tau}J(k)\right\} = \mathbb{E}\left\{\sum_{k=0}^{\tau}\left(e_z^{\mathrm{T}}(k)\mathcal{U}_1 e_z(k) + 2e_z^{\mathrm{T}}(k)\mathcal{U}_2\omega(k) + \omega^{\mathrm{T}}(k)\mathcal{U}_3\omega(k)\right)\right\} > \sup_{0\leq k\leq \tau}\mathbb{E}\left\{e_z^{\mathrm{T}}(k)\mathcal{U}_4 e_z(k)\right\}$$

To summarize, according to Definition 2, the error systems (8) are extended dissipative.

The proof is completed. ■

Remark 2. In the past efforts,[23] the quantization levels condition is given in the form of $\frac{1}{\mathcal{M}}\|y(\bar{k}_j)\| \leq \mu(k) \leq 2\eta\|y(\bar{k}_j)\|$, which may be unsolvable when $2\eta < \frac{1}{\mathcal{M}}$. In this paper, the dynamic quantizer is modified as Equation (14) which always exists on condition that the error systems satisfy Theorem 1. Nevertheless, the dynamic quantization levels have direct effect on the error systems performance. When the quantization levels are low, the performance degradation will be small at the cost that the amount of data to be transmitted will be large relatively. On the contrary, the amount of data to be transmitted will be small if the quantization levels are high, which leads to a large degradation of system performance. Therefore, appropriate quantization levels should be selected to balance the trade-off between performance degradation and the amount of data to be transmitted.

3.2 | Asynchronous filter design with the extended dissipative performance

Although a sufficient condition has been derived by Theorem 1 which ensures the stability and the desired extended dissipative performance, it is difficult to be used for filter design directly due to the time-varying MTPM and the membership function. In this subsection we focus on designing an asynchronous filter in the form of (7).

First of all, the following lemma is given, which is essential for further derivation.

Lemma 2. *For given membership functions satisfying* $g_j(\vartheta(k)) - l_j h_j(\vartheta(k)) \geq 0, (0 < l_j \leq 1)$ *and matrices* $E_{\zeta j} \in \mathbb{R}^{m\times n}$, $F_{\zeta j} \in \mathbb{R}^{p\times q}$, *if there exist symmetric matrices* $G^1_\zeta \in \mathbb{R}^{m\times n}, G^2_\zeta \in \mathbb{R}^{p\times q}$ *such that the following conditions hold for each* $\zeta, j \in \{1, 2, \dots, s\}$:

$$\begin{aligned} E_{\zeta j} - G^1_\zeta &< 0 \\ l_j E_{\zeta j} - l_j G^1_\zeta + G^1_\zeta &< 0 \\ F_{\zeta j} - G^2_\zeta &< 0 \\ l_j F_{\zeta j} - l_j G^2_\zeta + G^2_\zeta &< 0 \end{aligned} \tag{33}$$

then $\sum_{\zeta=1}^{s}\sum_{j=1}^{s} h_\zeta(\eta(k))g_j(\vartheta(k))E_{\zeta j} < 0$ *and* $\sum_{\zeta=1}^{s}\sum_{j=1}^{s} h_\zeta(\eta(k))g_j(\vartheta(k))F_{\zeta j} < 0$ *hold for all admissible grades of* h, g.

Proof. Similar to Kim et al.[27] slack symmetric matrices G^1_ζ, G^2_ζ are introduced. For $k \in \mathbb{N}$, there always exist

$$\begin{aligned} \sum_{\zeta=1}^{s}\sum_{j=1}^{s} h_\zeta(\eta(k)) \left[h_j(\vartheta(k)) - g_j(\vartheta(k))\right] G^1_\zeta &= \sum_{\zeta=1}^{s} h_\zeta(\eta(k)) G^1_\zeta \left[\sum_{j=1}^{s} h_j(\vartheta(k)) - \sum_{j=1}^{s} g_j(\vartheta(k))\right] \\ &= \sum_{\zeta=1}^{s} h_\zeta(\eta(k)) \left[1 - 1\right] G^1_\zeta \\ &= 0 \end{aligned} \tag{34}$$

$$\begin{aligned} \sum_{\zeta=1}^{s}\sum_{j=1}^{s} h_\zeta(\eta(k)) \left[h_j(\vartheta(k)) - g_j(\vartheta(k))\right] G^2_\zeta &= \sum_{\zeta=1}^{s} h_\zeta(\eta(k)) \left[1 - 1\right] G^2_\zeta \\ &= 0 \end{aligned} \tag{35}$$

Then considering the forms that

$$\sum_{\zeta=1}^{s}\sum_{j=1}^{s} h_\zeta(\eta(k)) g_j(\vartheta(k)) E_{\zeta j} \tag{36}$$

Combining (34) with (36), we have

$$\begin{aligned} \sum_{\zeta=1}^{s}\sum_{j=1}^{s} h_\zeta(\eta(k)) g_j(\vartheta(k)) E_{\zeta j} &= \sum_{\zeta=1}^{s}\sum_{j=1}^{s} h_\zeta(\eta(k)) \left[g_j(\vartheta(k)) E_{\zeta j} + \left(h_j(\vartheta(k)) - g_j(\vartheta(k))\right) G^1_\zeta\right] \\ &= \sum_{\zeta=1}^{s}\sum_{j=1}^{s} h_\zeta(\eta(k)) \left[h_j(\vartheta(k))(l_j E_{\zeta j} - l_j G^1_\zeta + G^1_\zeta) + (g_j(\vartheta(k)) - l_j h_j(\vartheta(k))(E_{\zeta j} - G^1_\zeta)\right] \end{aligned}$$

As membership functions satisfy $g_j(\vartheta(k)) - l_j h_j(\vartheta(k)) \geq 0, (0 < l_j \leq 1)$ and (33), we get

$$\sum_{\zeta=1}^{s}\sum_{j=1}^{s} h_\zeta(\eta(k)) g_j(\vartheta(k)) E_{\zeta j} < 0.$$

Similarly, we get

$$\sum_{\zeta=1}^{s}\sum_{j=1}^{s} h_\zeta(\eta(k)) g_j(\vartheta(k)) F_{\zeta j} < 0.$$

This completes the proof. ■

Noticing that Theorem 1 contains time-varying parameters and the membership function, it is difficult to get the filtering gain. For this reason, Theorem 2 is given which is independent of the membership function. Meanwhile, condition (24) is split into time-varying parts and time-invariant parts such that the asynchronous filter design can be completed.

Theorem 2. *For given matrices $\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \mathcal{U}_4$ satisfying Definition 2, a constant $\sigma > 0$, a positive number $\delta > 0$, scalars quantization range $\mathcal{M}$, error bound Δ and the membership functions satisfying $g_j(\vartheta(k)) - l_j h_j(\vartheta(k)) \geq 0$, where $0 < l_j \leq 1$, the error systems (8) are stochastically stable with a desired extended dissipative performance if there exist small enough scalar $\beta > 0$, matrices $P(a,m) > 0, \Phi_a > 0, A_{Fjm}, B_{Fjm}, C_{Fjm}$, invertible matrices $W^1_{jm}, W^2_{jm}, W^3_{jm}$, and symmetric matrices $\Gamma^1_\zeta, \Gamma^2_\zeta$ such that the following conditions hold for each $\zeta, j \in \{1,2,\dots,s\}$, $a \in \{1,2,\dots,s_1\}$ and $m \in \{1,2,\dots,s_2\}$:*

$$\begin{aligned} &\Lambda_{\zeta j} - \Gamma^1_\zeta < 0 \\ &l_j\Lambda_{\zeta j} - l_j\Gamma^1_\zeta + \Gamma^1_\zeta < 0 \end{aligned} \tag{37}$$

$$\begin{aligned} &Y^{\mathrm{T}}\tilde{P}(a,m)Y + \overline{\Omega}_{\zeta j}(a,m) - \Gamma^2_\zeta < 0 \\ &l_j(Y^{\mathrm{T}}\tilde{P}(a,m)Y + \overline{\Omega}_{\zeta j}(a,m)) - l_j\Gamma^2_\zeta + \Gamma^2_\zeta < 0 \end{aligned} \tag{38}$$

where

$$\overline{\Omega}_{\zeta j}(a,m) = \begin{bmatrix} \overline{\Omega}_1 & \overline{\Omega}_2 \\ * & \overline{\Omega}_3 \end{bmatrix}, \overline{\Omega}_1 = \begin{bmatrix} -P(a,m) + \sigma\vec{C}^{\mathrm{T}}_{\zeta a}\Phi_a\vec{C}_{\zeta a} & -\sigma\vec{C}^{\mathrm{T}}_{\zeta a}\Phi_a & 0 & -(\tilde{C}^F_{\zeta j})^{\mathrm{T}}\mathcal{U}_2 \\ * & (\sigma-1)\Phi_a + (\delta+1)^2 I & 0 & -\sigma D^{\mathrm{T}}_{\zeta a}\Phi_a \\ * & * & -\frac{\mathcal{M}^2}{\Delta^2}I & 0 \\ * & * & * & -\mathcal{H}e\{\tilde{D}^{\mathrm{T}}_{\zeta j}\mathcal{U}_2\} - \mathcal{U}_3 + \sigma D^{\mathrm{T}}_{\zeta a}\Phi_a D_{\zeta a} \end{bmatrix} + \beta I,$$

$$\overline{\Omega}_2 = \begin{bmatrix} \overline{\Omega}_{2,1} & (\delta+1)\vec{C}^{\mathrm{T}}_{\zeta a} & (\tilde{C}^F_{\zeta j})^{\mathrm{T}}(\mathcal{U}^+_1)^{\mathrm{T}} \\ -\overline{\Omega}_{2,2} & 0 & 0 \\ \overline{\Omega}_{2,2} & 0 & 0 \\ \overline{\Omega}_{2,3} & (\delta+1)D^{\mathrm{T}}_{\zeta a} & \tilde{D}^{\mathrm{T}}_{\zeta j}(\mathcal{U}^+_1)^{\mathrm{T}} \end{bmatrix}, \overline{\Omega}_3 = diag\left\{-W_{jm} - W^{\mathrm{T}}_{jm}, -I, -I\right\}, \Lambda_{\zeta j} = \begin{bmatrix} -P(a,m) & (\tilde{C}^F_{\zeta j})^{\mathrm{T}}(\mathcal{U}^+_4)^{\mathrm{T}} \\ * & -I \end{bmatrix},$$

$$W_{jm} = \begin{bmatrix} W^1_{jm} & W^2_{jm} \\ W^3_{jm} & W^2_{jm} \end{bmatrix}, \overline{\Omega}_{2,1} = \begin{bmatrix} \overline{\Omega}^1_{2,1} & A_{Fjm} \\ \overline{\Omega}^2_{2,1} & A_{Fjm} \end{bmatrix}^{\mathrm{T}}, \overline{\Omega}_{2,2} = \begin{bmatrix} -\alpha B^{\mathrm{T}}_{Fjm} & -\alpha B^{\mathrm{T}}_{Fjm} \end{bmatrix}, Y = \begin{bmatrix} 0 & 0 & 0 & 0 & I & 0 & 0 \end{bmatrix},$$

$$\overline{\Omega}_{2,3} = \begin{bmatrix} B^{\mathrm{T}}_{\zeta a}(W^1_{jm} + W^2_{jm})^{\mathrm{T}} - \alpha D^{\mathrm{T}}_{\zeta a}B^{\mathrm{T}}_{Fjm} & B^{\mathrm{T}}_{\zeta a}(W^3_{jm} + W^2_{jm})^{\mathrm{T}} - \alpha D^{\mathrm{T}}_{\zeta a}B^{\mathrm{T}}_{Fjm} \end{bmatrix}, \tilde{C}^F_{\zeta j} = \begin{bmatrix} L_{\zeta a} - C_{Fjm} & C_{Fjm} \end{bmatrix},$$

$$\overline{\Omega}^1_{2,1} = (W^1_{jm} + W^2_{jm})A_{\zeta a} - \alpha B_{Fjm}C_{\zeta a} - A_{Fjm}, \overline{\Omega}^2_{2,1} = (W^3_{jm} + W^2_{jm})A_{\zeta a} - \alpha B_{Fjm}C_{\zeta a} - A_{Fjm},$$

with $\vec{C}_{\zeta a} = [C_{\zeta a} \quad 0]$, $\tilde{P}(a,m) = \sum_{b=1}^{s_1}\sum_{n=1}^{s_2}\pi_{ab}(k)\varpi^b_{mn}(k)P(b,n)$, and the quantization levels condition is the same as (14). The gain matrices of the asynchronous filter are given by

$$A_{fjm} = (W^2_{jm})^{-1}A_{Fjm}, B_{fjm} = (W^2_{jm})^{-1}B_{Fjm}, C_{fjm} = C_{Fjm}. \tag{39}$$

Proof. Define invertible matrices W_m as follows

$$W_m = \sum_{j=1}^{s} g_j(\vartheta(k))W_{jm},$$

Substituting the gain matrices (39) into $\overline{\Omega}^1_{2,1}$ and $\overline{\Omega}^2_{2,1}$, there is

$$\begin{aligned} \overline{\Omega}^1_{2,1} &= (W^1_{jm} + W^2_{jm})A_{\zeta a} - \alpha W^2_{jm}B_{fjm}C_{\zeta a} - W^2_{jm}A_{fjm}, \\ \overline{\Omega}^2_{2,1} &= (W^3_{jm} + W^2_{jm})A_{\zeta a} - \alpha W^2_{jm}B_{fjm}C_{\zeta a} - W^2_{jm}A_{fjm}, \end{aligned}$$

Then, noticing $A_{Fjm} = W^2_{jm}A_{fjm}$ and the form of $\tilde{A}_{\zeta j}$ and W_{jm}, there is

$$\overline{\Omega}_{2,1} = \tilde{A}^{\mathrm{T}}_{\zeta j}W^{\mathrm{T}}_{jm}$$

Rewrite $\overline{\Omega}_{2,2}$, $\overline{\Omega}_{2,3}$ and $\tilde{C}^F_{\zeta j}$ as follows

$$\overline{\Omega}_{2,2} = \bar{E}_j^{\mathrm{T}} W_{jm}^{\mathrm{T}}, \quad \overline{\Omega}_{2,3} = \tilde{B}_{\zeta j}^{\mathrm{T}} W_{jm}^{\mathrm{T}}, \quad \tilde{C}^F_{\zeta j} = \tilde{C}_{\zeta j}, \tag{40}$$

Given that $\tilde{P}(a,m) > 0$, for $W_m = \sum_{j=1}^{s} g_j(\vartheta(k)) W_{jm}$, we have

$$\left(W_m - \tilde{P}(a,m)\right) \tilde{P}^{-1}(a,m) \left(W_m - \tilde{P}(a,m)\right)^{\mathrm{T}} \geq 0$$

Then there is

$$\tilde{P}(a,m) - W_m - W_m^{\mathrm{T}} \geq -W_m \tilde{P}^{-1}(a,m) W_m^{\mathrm{T}} \tag{41}$$

According to (40) and (41), one has

$$\sum_{\zeta=1}^{s}\sum_{j=1}^{s} h_\zeta(\eta(k)) g_j(\vartheta(k)) \left(Y^{\mathrm{T}} \tilde{P}(a,m) Y + \overline{\Omega}_{\zeta j}(a,m)\right) \geq \check{\Xi}$$

where

$$\check{\Xi} = \begin{bmatrix} \check{\Xi}_1 & \check{\Xi}_2 \\ * & \check{\Xi}_3 \end{bmatrix}, \check{\Xi}_1 = \hat{\Xi}_1, \check{\Xi}_2 = \begin{bmatrix} \tilde{A}_{ha}^{\mathrm{T}} W_m^{\mathrm{T}} & (\delta+1)\vec{C}_{ha}^{\mathrm{T}} & \tilde{C}_{ha}^{\mathrm{T}} (\mathcal{U}_1^+)^{\mathrm{T}} \\ -\bar{E}_{ha}^{\mathrm{T}} W_m^{\mathrm{T}} & 0 & 0 \\ \bar{E}_{ha}^{\mathrm{T}} W_m^{\mathrm{T}} & 0 & 0 \\ \tilde{B}_{ha}^{\mathrm{T}} W_m^{\mathrm{T}} & (\delta+1) D_{ha}^{\mathrm{T}} & \tilde{D}_{ha}^{\mathrm{T}} (\mathcal{U}_1^+)^{\mathrm{T}} \end{bmatrix}, \check{\Xi}_3 = diag\left\{-W_m \tilde{P}^{-1}(a,m) W_m^{\mathrm{T}}, -I, -I\right\}$$

in which $\hat{\Xi}_1$ is defined in (24). Since the membership functions satisfy $g_j(\vartheta(k)) - l_j h_j(\vartheta(k)) \geq 0, (0 < l_j \leq 1)$, applying Lemma 2 for (37) and (38), it holds that

$$\sum_{\zeta=1}^{s}\sum_{j=1}^{s} h_\zeta(\eta(k)) g_j(\vartheta(k)) \Lambda_{\zeta j} = \Lambda < 0$$

$$\check{\Xi} \leq \sum_{\zeta=1}^{s}\sum_{j=1}^{s} h_\zeta(\eta(k)) g_j(\vartheta(k)) \left(Y^{\mathrm{T}} \tilde{P}(a,m) Y + \overline{\Omega}_{\zeta j}(a,m)\right) < 0$$

Then (25) is satisfied.

Multiplying $\check{\Xi}$ by $\overline{W}_m$ and $\overline{W}_m^{\mathrm{T}}$ on the left-hand side and right-hand side, respectively, where $\overline{W}_m = diag\{I, I, I, I, \tilde{P}(a,m) W_m^{-1}, I, I\}$, since $\check{\Xi} < 0$, it yields that

$$\overline{W}_m \check{\Xi} \overline{W}_m^{\mathrm{T}} = \hat{\Xi} < 0$$

Then (24) is satisfied.

Since both (24) and (25) are satisfied, according to Theorem 1, the error systems (8) are extended dissipative. Meanwhile, (24) is the sufficient condition for (13) because of the properties of negative definite matrix.[31] Therefore, the noise-free error systems (11) are stochastically stable. To summarize, the error systems (8) are stochastically stable with a desired extended dissipative performance.

This completes the proof. ■

Remark 3. To the best of our knowledge, most existing achievements require that the premise variables of FN-MJSs are available to the filter.[32,33] However, the asynchronous filter (7) may have difficulties in obtaining the perfect matched premise variables from the FN-MJSs (1) due to the package dropout. In this paper, we take into account a more general scenario that the premise variables of FN-MJSs $\eta(k)$ are unavailable to the asynchronous filter, which makes our filter design more feasible. In Theorem 2, the membership functions satisfy $g_j(\vartheta(k)) - l_j h_j(\vartheta(k)) \geq 0$, with $0 < l_j \leq 1$. It is easy to see that such l_j is sure to exist on the basis that the growth rate of the membership function is less than or equal to the linear growth rate.

Theorem 2 still includes the time-varying MTPM, which means we have to solve infinite linear matrix inequalities to obtain the filter gain. To deal with this, the following theorem proposes a sufficient condition for the existence of the asynchronous filter with the prescribed extended dissipative performance. The conditions in this theorem are time-invariant such that the linear matrix inequalities are solvable.

Theorem 3. *For given matrices $\mathcal{U}_1$, $\mathcal{U}_2$, $\mathcal{U}_3$, $\mathcal{U}_4$ satisfying Definition 2, scalars quantization range $\mathcal{M}$, error bound Δ, a positive number $\delta > 0$, a constant $\sigma > 0$ and the membership functions satisfying $g_j(\vartheta(k)) - l_j h_j(\vartheta(k)) \geq 0$, where $0 < l_j \leq 1$, the error systems (8) are stochastically stable with a desired extended dissipative performance, if there exist small enough scalar $\beta > 0$, matrices $P(a,m) > 0$, $\Phi_a > 0$, $\overline{\phi}^1_{bmn} < 0$, $\overline{\phi}^2_{ab} < 0$, $\overline{\phi}^4_{bm}$, $\overline{\phi}^5_{bm}$, A_{Fjm}, B_{Fjm}, C_{Fjm}, invertible matrices W^1_{jm}, W^2_{jm}, W^3_{jm}, and symmetric matrices Γ^1_ζ, Γ^2_ζ such that the following conditions hold for each $\zeta, j \in \{1, 2, \ldots, s\}$, $a \in \{1, 2, \ldots, s_1\}$ and $m \in \{1, 2, \ldots, s_2\}$:*

$$\Upsilon^1_{\zeta j} + \Upsilon < 0 \tag{42}$$

$$\Upsilon^2_{\zeta j} + l_j \Upsilon < 0 \tag{43}$$

$$\begin{aligned} &\Lambda_{\zeta j} - \Gamma^1_\zeta < 0 \\ &l_j \Lambda_{\zeta j} - l_j \Gamma^1_\zeta + \Gamma^1_\zeta < 0 \end{aligned} \tag{44}$$

where

$$\Upsilon^1_{\zeta j} = \left[\begin{array}{c|c} \bar{\Omega}_{\zeta j}(a,m) - \Gamma^2_\zeta & 0 \\ \hline 0 & 0 \end{array}\right], \Upsilon^2_{\zeta j} = \left[\begin{array}{c|c} l_j\left(\bar{\Omega}_{\zeta j}(a,m) - \Gamma^2_\zeta\right) + \Gamma^2_\zeta & 0 \\ \hline 0 & 0 \end{array}\right],$$

$$\Upsilon = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5, \bar{\phi}^3_{bn} = P(b,n),$$

$$\phi_1 = \left[\begin{array}{c|ccc} \sum_{b\in\mathcal{S}_1} \sum_{n\in\mathcal{S}^n_V} \breve{\varpi}^b_{mn} \hat{\varpi}^b_{mn} Y^{\mathrm{T}} \mathcal{H}e\{\bar{\phi}^1_{bmn}\} Y & * & * & * \\ \hline 0 & 0 & * & * \\ \left[\left[-\bar{\varpi}^b_{mn} \bar{\phi}^1_{bmn} Y\right]_{n\in\mathcal{S}^n_V}\right]_{b\in\mathcal{S}^b_V} & 0 & \left[\left[\mathcal{H}e\{\bar{\phi}^1_{bmn}\}\right]^D_{n\in\mathcal{S}^n_V}\right]^D_{b\in\mathcal{S}^b_V} & * \\ \left[\left[-\bar{\varpi}^b_{mn} \bar{\phi}^1_{bmn} Y\right]_{n\in\mathcal{S}^n_V}\right]_{b\in\mathcal{S}^b_{IV}} & 0 & 0 & \left[\left[\mathcal{H}e\{\bar{\phi}^1_{bmn}\}\right]^D_{n\in\mathcal{S}^n_V}\right]^D_{b\in\mathcal{S}^b_{IV}} \end{array}\right],$$

$$\phi_2 = \left[\begin{array}{c|ccc} \sum_{b\in\mathcal{S}^b_V} \breve{\pi}_{ab} \hat{\pi}_{ab} Y^{\mathrm{T}} \mathcal{H}e\{\bar{\phi}^2_{ab}\} Y & * & * & * \\ \hline \left[-\bar{\pi}_{ab} \bar{\phi}^2_{ab} Y\right]_{b\in\mathcal{S}^b_V} & \left[\mathcal{H}e\{\bar{\phi}^2_{ab}\}\right]^D_{b\in\mathcal{S}^b_V} & * & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 \end{array}\right],$$

$$\phi_3 = \left[\begin{array}{c|ccc} \sum_{b\in\mathcal{S}^b_{IV}} \sum_{n\in\mathcal{S}^n_{IV}} \varpi^b_{mn} \pi_{ab} Y^{\mathrm{T}} \bar{\phi}^3_{bn} Y & * & * & * \\ \hline \left[\sum_{n\in\mathcal{S}^n_{IV}} \frac{\varpi^b_{mn}}{2} \bar{\phi}^3_{bn} Y\right]_{b\in\mathcal{S}^b_V} & 0 & * & * \\ 0 & \left[\left[\frac{1}{2}\bar{\phi}^3_{bn}\right]_{n\in\mathcal{S}^n_V}\right]^D_{b\in\mathcal{S}^b_V} & 0 & * \\ \left[\left[\frac{\pi_{ab}}{2} \bar{\phi}^3_{bn} Y\right]_{n\in\mathcal{S}^n_V}\right]_{b\in\mathcal{S}^b_{IV}} & 0 & 0 & 0 \end{array}\right],$$

$$\phi_4 = \begin{bmatrix} \sum_{b\in\mathcal{S}_{IV}^b}\pi_{ab}(\Psi_{IV}-1)Y^{\mathrm{T}}\mathcal{H}e\{\bar{\phi}_{bm}^4\}Y & * & * & * \\ \left[(\Psi_{IV}-1)\bar{\phi}_{bm}^4 Y\right]_{b\in\mathcal{S}_V^b} & 0 & * & * \\ 0 & \left[[\bar{\phi}_{bm}^4]_{n\in\mathcal{S}_V^n}\right]_{b\in\mathcal{S}_V^b}^D & 0 & * \\ \left[\left[\pi_{ab}\bar{\phi}_{bm}^4 Y\right]_{n\in\mathcal{S}_V^n}\right]_{b\in\mathcal{S}_{IV}^b} & 0 & 0 & 0 \end{bmatrix},$$

$$\phi_5 = \begin{bmatrix} Y^{\mathrm{T}}\mathcal{H}e\left\{(\Pi_{IV}-1)\bar{\phi}_a^5\right\}Y & * & * & * \\ \left[\bar{\phi}_a^5 Y\right]_{b\in\mathcal{S}_V^b} & 0 & * & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

with $\vec{C}_{\zeta a} = [C_{\zeta a} \quad 0]$. *Dynamic quantization levels* $\mu(k)$ *are the same as (14).* $\Psi_{IV}, \Pi_{IV}, \overline{\varpi}_{mn}^b$ *and* $\overline{\pi}_{ab}$ *are defined in (10).* Y, $\Lambda_{\zeta j}$, $\overline{\Omega}_{\zeta j}(a, m)$ *and the gain matrices of the asynchronous filter are defined in Theorem 2.*

Proof. Define Θ as the follows,

$$\Theta \triangleq Y^{\mathrm{T}}\tilde{P}(a, m)Y = \sum_{b=1}^{s_1}\sum_{n=1}^{s_2}\pi_{ab}(k)\varpi_{mn}^b(k)Y^{\mathrm{T}}P(b, n)Y$$

Then, we define $\phi_{3,11} = \sum_{b\in\mathcal{S}_{IV}^b}\sum_{n\in\mathcal{S}_{IV}^n}\varpi_{mn}^b\pi_{ab}Y^{\mathrm{T}}\overline{\phi}_{bn}^3 Y$. Considering whether the transition probabilities in MTPM are time varying, we have

$$\begin{aligned}\Theta = &\sum_{b\in\mathcal{S}_{IV}^b}\sum_{n\in\mathcal{S}_V^n}\varpi_{mn}^b(k)\pi_{ab}Y^{\mathrm{T}}\overline{\phi}_{bn}^3 Y + \sum_{b\in\mathcal{S}_V^b}\sum_{n\in\mathcal{S}_{IV}^n}\varpi_{mn}^b\pi_{ab}(k)Y^{\mathrm{T}}\overline{\phi}_{bn}^3 Y \\ &+ \sum_{b\in\mathcal{S}_V^b}\sum_{n\in\mathcal{S}_V^n}\varpi_{mn}^b(k)\pi_{ab}(k)Y^{\mathrm{T}}\overline{\phi}_{bn}^3 Y + \phi_{3,11}\end{aligned}$$

Since $\varpi_{mn}^b(k)$ and $\pi_{ab}(k)$ are scalars, we change their positions in the equation and get

$$\begin{aligned}\Theta = \mathcal{H}e\Bigg\{&\sum_{b\in\mathcal{S}_{IV}^b}\sum_{n\in\mathcal{S}_V^n}\varpi_{mn}^b(k)Y^{\mathrm{T}}\frac{1}{2}\pi_{ab}\overline{\phi}_{bn}^3 Y + \sum_{b\in\mathcal{S}_V^b}\left(\pi_{ab}(k)Y^{\mathrm{T}}\sum_{n\in\mathcal{S}_{IV}^n}\frac{1}{2}\varpi_{mn}^b\overline{\phi}_{bn}^3 Y\right) \\ &+ \sum_{b\in\mathcal{S}_V^b}\sum_{n\in\mathcal{S}_V^n}\varpi_{mn}^b(k)Y^{\mathrm{T}}\frac{1}{2}\overline{\phi}_{bn}^3\pi_{ab}(k)Y\Bigg\} + \phi_{3,11}\end{aligned}$$

Motivated by the method of Kim et al.[15] by the form of Kronecker product, Θ can be rewritten as

$$\begin{aligned}\Theta = \mathcal{H}e\Bigg\{&\sum_{b\in\mathcal{S}_{IV}^b}[\Psi_V\otimes Y]^{\mathrm{T}}[\frac{1}{2}\pi_{ab}\overline{\phi}_{bn}^3 Y]_{n\in\mathcal{S}_V^n} + [\Pi_V\otimes Y]^{\mathrm{T}}[\sum_{n\in\mathcal{S}_{IV}^n}\frac{1}{2}\varpi_{mn}^b\overline{\phi}_{bn}^3 Y]_{b\in\mathcal{S}_V^b} \\ &+ \sum_{b\in\mathcal{S}_V^b}[\Psi_V\otimes Y]^{\mathrm{T}}[\frac{1}{2}\overline{\phi}_{bn}^3]_{n\in\mathcal{S}_V^n}[\pi_{ab}(k)Y]\Bigg\} + \phi_{3,11}\end{aligned}$$

Furthermore, we consider the other summation term of Θ, we have

$$\begin{aligned}\Theta = \mathcal{H}e\Bigg\{&[\Psi_V\otimes Y]_{b\in\mathcal{S}_{IV}^b}^{\mathrm{T}}[[\frac{1}{2}\pi_{ab}\overline{\phi}_{bn}^3 Y]_{n\in\mathcal{S}_V^n}]_{b\in\mathcal{S}_{IV}^b} + [\Pi_V\otimes Y]^{\mathrm{T}}\left[\sum_{n\in\mathcal{S}_{IV}^n}\frac{1}{2}\varpi_{mn}^b\overline{\phi}_{bn}^3 Y\right]_{b\in\mathcal{S}_V^b} \\ &+[\Psi_V\otimes Y]_{b\in\mathcal{S}_V^b}^{\mathrm{T}}[[\frac{1}{2}\overline{\phi}_{bn}^3]_{n\in\mathcal{S}_V^n}]_{b\in\mathcal{S}_V^b}^D[\Pi_V\otimes Y]\Bigg\} + \phi_{3,11}\end{aligned} \quad (45)$$

Let ϱ be of the following form:

$$\varrho = \begin{bmatrix} I & [\Pi_V \otimes Y]^{\mathrm{T}} & [\Psi_V \otimes Y]^{\mathrm{T}}_{b\in\mathcal{S}_V^b} & [\Psi_V \otimes Y]^{\mathrm{T}}_{b\in\mathcal{S}_{IV}^b} \end{bmatrix}^{\mathrm{T}} \tag{46}$$

then combining (45) with (46), we have

$$\Theta = \varrho^{\mathrm{T}}\phi_3\varrho \tag{47}$$

In addition, since $\mathcal{H}e\{\overline{\phi}^1_{bmn}\} < 0$ and $\mathcal{H}e\{\overline{\phi}^2_{ab}\} < 0$, based on the MTPM's boundary (9), we have the following inequalities:

$$\sum_{b\in\mathcal{S}_1}\sum_{n\in\mathcal{S}_V^n}(\varpi^b_{mn}(k) - \check{\varpi}^b_{mn})(\varpi^b_{mn}(k) - \hat{\varpi}^b_{mn})Y^{\mathrm{T}}\mathcal{H}e\{\overline{\phi}^1_{bmn}\}Y > 0 \tag{48}$$

$$\sum_{b\in\mathcal{S}_V^b}(\pi_{ab}(k) - \check{\pi}_{ab})(\pi_{ab}(k) - \hat{\pi}_{ab})Y^{\mathrm{T}}\mathcal{H}e\{\overline{\phi}^2_{ab}\}Y > 0 \tag{49}$$

Similarly, (48), (49) yields

$$\varrho^{\mathrm{T}}\phi_1\varrho > 0, \qquad \varrho^{\mathrm{T}}\phi_2\varrho > 0 \tag{50}$$

Considering the condition $\sum_{n=1}^{S_2}\varpi^{r_{k+1}}_{mn}(k) = 1$, one has

$$\left(\sum_{n\in\mathcal{S}_{IV}^n}\varpi^b_{mn} - 1\right) + \sum_{n\in\mathcal{S}_V^n}\varpi^b_{mn}(k) = (\Psi_{IV} - 1) + \sum_{n\in\mathcal{S}_V^n}\varpi^b_{mn}(k) = 0,$$

where Ψ_{IV} is defined in (10). Furthermore, we can rewrite it as

$$\begin{aligned} 0 = &\sum_{b\in\mathcal{S}_{IV}^b}\pi_{ab}\left((\Psi_{IV} - 1) + \sum_{n\in\mathcal{S}_V^n}\varpi^b_{mn}(k)\right)Y^{\mathrm{T}}\mathcal{H}e\{\overline{\phi}^4_{bm}\}Y + \sum_{b\in\mathcal{S}_V^b}\pi_{ab}(k)\left((\Psi_{IV} - 1) + \sum_{n\in\mathcal{S}_V^n}\varpi^b_{mn}(k)\right)Y^{\mathrm{T}}\mathcal{H}e\{\overline{\phi}^4_{bm}\}Y \\ = &\sum_{b\in\mathcal{S}_{IV}^b}\pi_{ab}(\Psi_{IV} - 1)Y^{\mathrm{T}}\mathcal{H}e\{\overline{\phi}^4_{bm}\}Y + \mathcal{H}e\left\{\sum_{b\in\mathcal{S}_{IV}^b}\sum_{n\in\mathcal{S}_V^n}\varpi^b_{mn}(k)Y^{\mathrm{T}}\pi_{ab}\overline{\phi}^4_{bm}Y\right\} \\ &+ \mathcal{H}e\left\{\sum_{b\in\mathcal{S}_V^b}\pi_{ab}(k)Y^{\mathrm{T}}(\Psi_{IV} - 1)\overline{\phi}^4_{bm}Y\right\} + \mathcal{H}e\left\{\sum_{b\in\mathcal{S}_V^b}\sum_{n\in\mathcal{S}_V^n}\pi_{ab}(k)\varpi^b_{mn}(k)Y^{\mathrm{T}}\overline{\phi}^4_{bm}Y\right\} \end{aligned} \tag{51}$$

Combining (51) with (46), we have

$$\varrho^{\mathrm{T}}\phi_4\varrho = 0 \tag{52}$$

By the similar process, bearing in mind that $\sum_{b=1}^{S_1}\pi_{ab}(k) = 1$, there is

$$\varrho^{\mathrm{T}}\phi_5\varrho = 0 \tag{53}$$

Combining (47), (50), (52) with (53), one has

$$\varrho^{\mathrm{T}}(\phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5)\varrho = \varrho^{\mathrm{T}}\Upsilon\varrho > \varrho^{\mathrm{T}}\phi_3\varrho = \Theta$$

Considering the form of ϱ and Θ, according to condition (42) and (43), we deduce that

$$0 > \varrho^{\mathrm{T}}(\Upsilon^1_{\zeta j} + \Upsilon)\varrho > \Theta + \overline{\Omega}_{\zeta j}(a, m) - \Gamma^2_\zeta = Y^{\mathrm{T}}\tilde{P}(a, m)Y + \overline{\Omega}_{\zeta j}(a, m) - \Gamma^2_\zeta$$

$$0 > \varrho^{\mathrm{T}}(\Upsilon^2_{\zeta j} + l_j\Upsilon)\varrho > l_j\Theta + l_j(\overline{\Omega}_{\zeta j}(a, m) - \Gamma^2_\zeta) + \Gamma^2_\zeta = l_j\left(Y^{\mathrm{T}}\tilde{P}(a, m)Y + \overline{\Omega}_{\zeta j}(a, m)\right) - l_j\Gamma^2_\zeta + \Gamma^2_\zeta$$

Then (38) is satisfied. Meanwhile, (44) is the same as (37) in Theorem 2. According to Theorem 2, the error systems (8) are stochastically stable with a desired extended dissipative performance.

This completes the proof. ■

Remark 4. Theorem 3 presents the asynchronous filter design for the networked FN-MJSs such that the extended dissipative performance can be satisfied. And this performance provides more flexibility with specific performance parameters $\mathcal{U}_1$, $\mathcal{U}_2$, $\mathcal{U}_3$ and $\mathcal{U}_4$ taken, which covers the H_∞ performance, $l_2 - l_\infty$ performance, passivity and dissipativity. To be specific, by setting $\mathcal{U}_1 = -I$, $\mathcal{U}_2 = 0$, $\mathcal{U}_3 = \gamma^2 I$ and $\mathcal{U}_4 = 0$, Theorem 3 turns into H_∞ filtering case, which has been studied in Hua et al.[24] The strict (Q, S, R)-dissipativity filtering can be obtained if one choose $\mathcal{U}_1 = Q$, $\mathcal{U}_2 = S$, $\mathcal{U}_3 = R - \gamma I$ and $\mathcal{U}_4 = 0$, which covers the existing results of Kim et al.[15] Meanwhile, the non-homogeneous filtering considered in this paper is a more general situation. When the MTPM of MJSs (1) is time-invariant, Theorem 3 turns into the results in Tao et al.[22]

4 | SIMULATION

In this section, a practical example is presented to demonstrate the effectiveness of the proposed event-triggered asynchronous filter design with dynamic quantization for the networked FN-MJSs.

Considering a tunnel diode circuit shown in Figure 2, it fuzzy model is given by Ding et al.[34] as

$$i_D(t) = 0.002V_D(t) + 0.01V_D^3(t)$$

where $x_1(t) \triangleq V_C(t), x_2(t) \triangleq i_L(t)$. The tunnel diode circuit can be described by the following equations:

$$\begin{cases} C\dot{x}_1(t) = -0.002x_1(t) - 0.01x_1^3(t) + x_2(t) \\ L\dot{x}_2(t) = -x_1(t) - Rx_2(t) + \omega(t) \\ y(t) = x_1(t) \\ z(t) = x_1(t) + 0.1\omega(t) \end{cases}$$

The parameters in the circuit are chosen as: the resistance $R = 1\Omega$, the capacitance $C = 20$ mF and the inductance $L = 1$ H. By the Euler's discretization method, the aforementioned equations can be rewritten as:

$$\begin{cases} x_1(k+1) = x_1(k) + T(-0.5x_1^3(k) - 0.1x_1(k) + 50x_2(k)) \\ x_2(k+1) = x_2(k) + T(-x_1(k) - x_2(k) + \omega(k)) \\ y(k) = x_1(k) \\ z(k) = x_1(k) + 0.1\omega(k) \end{cases}$$

where T is the sampling time. Similarity, the fuzzy modeling of the other mode are the same as Assawinchaichote et al.[3] Using a sampling time $T = 0.02s$, the discrete-time T-S fuzzy model is obtained as:

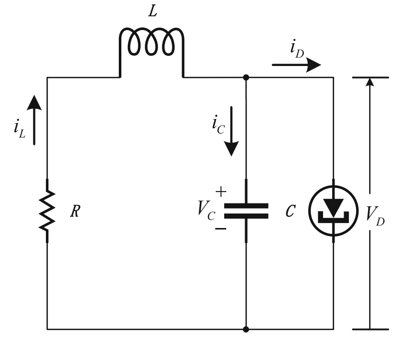

FIGURE 2 Tunnel diode circuit

Plant Rule ζ: IF $x_1(k)$ is $h_\zeta(x_1(k))$, THEN

$$\begin{cases} x(k+1) = A_{\zeta a}x(k) + B_{\zeta a}\omega(k) \\ y(k) = C_{\zeta a}x(k) \\ z(k) = L_{\zeta a}x(k) + R_{\zeta a}\omega(k) \end{cases}$$

where the parameters are given as: for all ζ,a,

$$A_{11} = \begin{bmatrix} 0.9987 & 0.9024 \\ -0.0180 & 0.8100 \end{bmatrix}, A_{12} = \begin{bmatrix} 0.998 & 1 \\ -0.02 & 0.98 \end{bmatrix}, A_{21} = \begin{bmatrix} 0.90337 & 0.8617 \\ -0.0172 & 0.8103 \end{bmatrix},$$
$$A_{22} = \begin{bmatrix} 0.908 & 1 \\ -0.02 & 0.98 \end{bmatrix}, B_{11} = \begin{bmatrix} 0.0093 \\ 0.0181 \end{bmatrix}, B_{12} = \begin{bmatrix} 0 \\ 0.02 \end{bmatrix}, B_{21} = \begin{bmatrix} 0.0091 \\ 0.0181 \end{bmatrix},$$
$$B_{22} = \begin{bmatrix} 0 \\ 0.02 \end{bmatrix}, C_{\zeta a} = \begin{bmatrix} 1 & 0 \end{bmatrix}, L_{\zeta a} = \begin{bmatrix} 1 & 0 \end{bmatrix}, R_{\zeta a} = 0.1,$$

The membership functions of the FN-MJSs are given as:

$$h_1(x_1(k)) = \left(1 - \frac{1}{1+e^{-3(x_1(k)-0.5\pi)}}\right)\left(\frac{1}{1+e^{-3(x_1(k)+0.5\pi)}}\right)$$
$$h_2(x_1(k)) = 1 - h_1(x_1(k))$$

And the membership functions of the filter are chosen as:

$$g_1(\hat{x}_1(k)) = 0.99e^{\frac{-\hat{x}_1^2(k)}{2\times 1.5^2}}, \qquad g_2(\hat{x}_1(k)) = 1 - g_1(\hat{x}_1(k))$$

According to $g_j(\hat{x}_1(k)) - l_j h_j(\hat{x}_1(k)) \geq 0$, we can get $l_1 = 0.8, l_2 = 0.95$. Here we assume the time-varying MTPMs are totally unknown whose bounds are given by:

$$\check{\pi}_{ab} = \begin{bmatrix} 0.5 & 0.2 \\ 0.4 & 0.3 \end{bmatrix}, \hat{\pi}_{ab} = \begin{bmatrix} 0.8 & 0.5 \\ 0.7 & 0.6 \end{bmatrix}, \check{\varpi}^1_{mn} = \begin{bmatrix} 0.6 & 0.3 \\ 0.6 & 0.3 \end{bmatrix},$$
$$\hat{\varpi}^1_{mn} = \begin{bmatrix} 0.7 & 0.4 \\ 0.7 & 0.4 \end{bmatrix}, \check{\varpi}^2_{mn} = \begin{bmatrix} 0 & 0.8 \\ 0 & 0.8 \end{bmatrix}, \hat{\varpi}^2_{mn} = \begin{bmatrix} 0.2 & 1 \\ 0.2 & 1 \end{bmatrix},$$

Let the initial system state be $x(0) = \begin{bmatrix} -2.5 & 1 \end{bmatrix}^T$ while the initial state of the filter is $\hat{x}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. The expectation of $\alpha_{\bar{k}_j}$ can be selected as $\mathbb{E}\{\alpha_{\bar{k}_j}\} = 0.8$. And the disturbance noise is borrowed from Liu et al.,[35] that is, $\omega(k) = sin(k)e^{-0.1k}$. Now taking into account the design of the event-triggered asynchronous filter with dynamic quantization, the event-triggered scheme parameter is set as $\sigma = 0.1$. And motivated by Liu et al.,[36] the dynamic quantizer is chosen as

$$q_\mu(y(\bar{k}_j)) = \begin{cases} 100\mu sgn(y(\bar{k}_j)), & if \|y(\bar{k}_j)\| > 100\mu \\ \mu\lfloor \frac{y(\bar{k}_j)}{\mu} + 0.1 \rfloor, & if \|y(\bar{k}_j)\| \leq 100\mu \end{cases}$$

where the quantization ranges are defined as $\mathcal{M} = 100$, the error bounds are given as $\Delta = 0.1$. According to Lemma 1, the quantization levels parameter is $\delta = 0.0514$. From (14), we obtain the quantization levels condition:

$$1 \times 10^{-2}\|y(\bar{k}_j)\| \leq \mu(k) \leq 1.0514 \times 10^{-2}\|y(\bar{k}_j)\|$$

Without loss of generality, we choose $\mu(k) = 1.0514 \times 10^{-2}\|y(\bar{k}_j)\|$.

In this paper, two classical performance are investigated: H_∞ performance and $l_2 - l_\infty$ performance which are included in the extended dissipative performance.

4.1 | Case I: H_∞ performance

Firstly we define the extended dissipative performance parameters as $\mathcal{U}_1 = -I$, $\mathcal{U}_2 = 0$, $\mathcal{U}_3 = \gamma^2 I$, $\mathcal{U}_4 = 0$, and this corresponds to the H_∞ performance. According to Theorem 3, we can obtain the H_∞ filter gains and the event-triggered parameters:

$$A_{f11} = \begin{bmatrix} -0.1171 & 1.2549 \\ -0.1460 & 0.9811 \end{bmatrix}, A_{f12} = \begin{bmatrix} -0.0985 & 1.1263 \\ -0.1422 & 0.9631 \end{bmatrix}, A_{f21} = \begin{bmatrix} -0.1171 & 1.2549 \\ -0.1460 & 0.9811 \end{bmatrix}, A_{f22} = \begin{bmatrix} -0.0985 & 1.1279 \\ -0.1422 & 0.9631 \end{bmatrix},$$

$$B_{f11} = \begin{bmatrix} 2.4242 \\ 0.2855 \end{bmatrix}, B_{f12} = \begin{bmatrix} 2.3750 \\ 0.2746 \end{bmatrix}, B_{f21} = \begin{bmatrix} 2.4242 \\ 0.2855 \end{bmatrix}, B_{f22} = \begin{bmatrix} 2.3750 \\ 0.2746 \end{bmatrix}, \Phi_1 = 4.3488, \Phi_2 = 4.3227,$$

$$C_{f11} = \begin{bmatrix} 0.5087 & 0.0686 \end{bmatrix}, C_{f12} = \begin{bmatrix} 0.5008 & 0.2519 \end{bmatrix}, C_{f21} = \begin{bmatrix} 0.5087 & 0.0686 \end{bmatrix}, C_{f22} = \begin{bmatrix} 0.5008 & 0.2519 \end{bmatrix},$$

The optimal noise attenuation performance is $\gamma^* = 2.0386$. Consequently, we can have a conclusion that the error systems (8) are stochastically stable with prescribed H_∞ noise attenuation performance $\gamma^* = 2.0386$.

With the above parameters, the Monte Carlo simulation is performed and we run this simulation 100,000 times to account for the average H_∞ performance under dropouts. Figure 3 depicts the systems and filter mode, which shows that the filter mode is asynchronous with the original one. Figure 4 displays the objective signal and average estimated signal(100,000 times) with dynamic quantization where the red dotted line and blue line denote the $z(k)$ and $\mathbb{E}\{\hat{z}(k)\}$ respectively. Here the pink area represents the location of all 100,000 estimated signals. Figure 5 shows the error response where blue line denotes $\mathbb{E}\{e_z(k)\}$ and the pink area represents the location of all 100,000 error responses. One sample of the release interval shown in Figure 6 implies that the event-triggered scheme is effective to reduce the sampling frequency. The dynamic quantization levels shown in Figure 7 are time-varying and get small gradually to mitigate performance degradation.

4.2 | Case II: $l_2 - l_\infty$ performance

Here we choose the extended dissipative performance parameters as $\mathcal{U}_1 = 0$, $\mathcal{U}_2 = 0$, $\mathcal{U}_3 = \gamma^2 I$, $\overline{R}_h = 0$, $\mathcal{U}_4 = I$ and this corresponds to the $l_2 - l_\infty$ performance. According to Theorem 3, we can obtain $l_2 - l_\infty$ filter gains and the event-triggered parameters:

$$A_{f11} = \begin{bmatrix} -0.0508 & 1.2668 \\ -0.1452 & 0.9860 \end{bmatrix}, A_{f12} = \begin{bmatrix} -0.0193 & 1.1543 \\ -0.1391 & 0.9659 \end{bmatrix}, A_{f21} = \begin{bmatrix} -0.0508 & 1.2667 \\ -0.1452 & 0.9860 \end{bmatrix}, A_{f22} = \begin{bmatrix} -0.0193 & 1.1542 \\ -0.1391 & 0.9659 \end{bmatrix},$$

$$B_{f11} = \begin{bmatrix} 2.2396 \\ 0.2787 \end{bmatrix}, B_{f12} = \begin{bmatrix} 2.1625 \\ 0.2631 \end{bmatrix}, B_{f21} = \begin{bmatrix} 2.2396 \\ 0.2787 \end{bmatrix}, B_{f22} = \begin{bmatrix} 2.1625 \\ 0.2631 \end{bmatrix}, \Phi_1 = 3.2709, \Phi_2 = 3.2626,$$

$$C_{f11} = \begin{bmatrix} 0.1421 & -0.8163 \end{bmatrix}, C_{f12} = \begin{bmatrix} 0.1425 & -0.8205 \end{bmatrix}, C_{f21} = \begin{bmatrix} 0.1419 & -0.8158 \end{bmatrix}, C_{f22} = \begin{bmatrix} 0.1422 & -0.8191 \end{bmatrix},$$

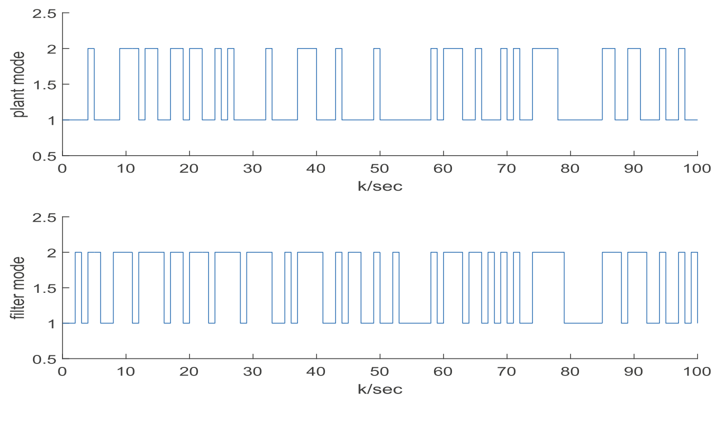

FIGURE 3 The evolution of systems and filter mode of H_∞ performance

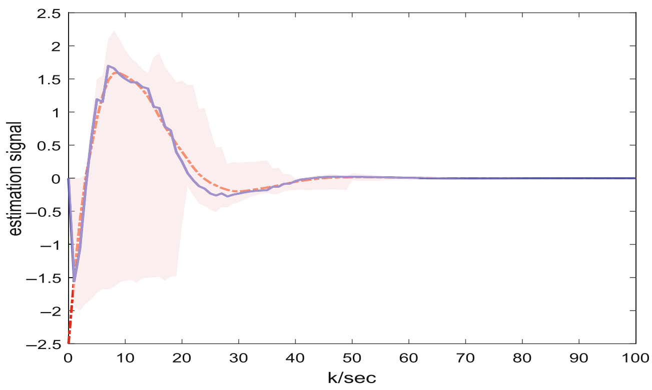

FIGURE 4 The estimation signal of systems and filter of H_∞ performance

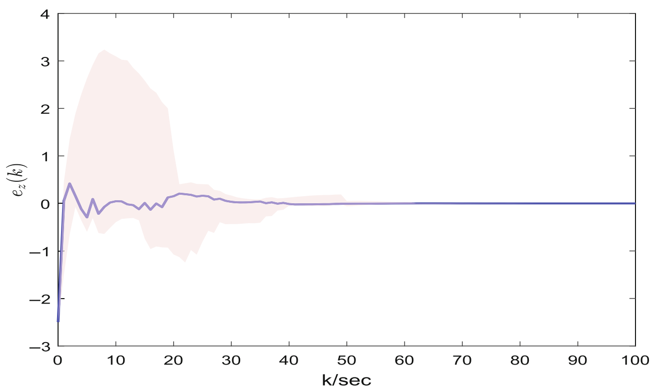

FIGURE 5 Error response of H_∞ performance

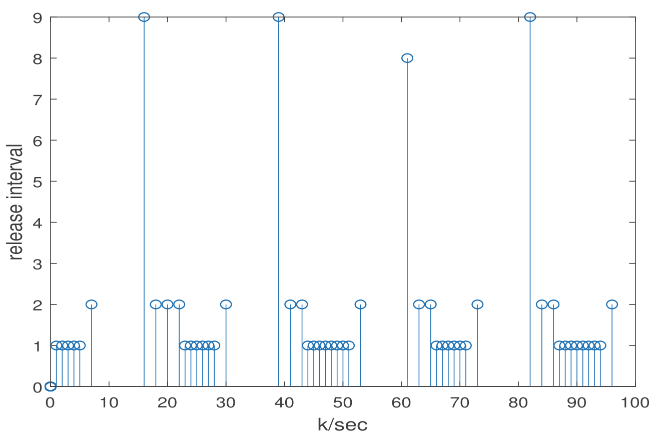

FIGURE 6 One sample of release interval and instants of H_∞ performance

The optimal $l_2 - l_\infty$ noise attenuation performance $\gamma^* = 1.9552$. Consequently, the error systems (8) under quantization levels condition are stochastically stable with an $l_2 - l_\infty$ disturbance attenuation performance.

Similarly, the Monte Carlo simulation is performed and we run this simulation 100,000 times to account for the average $l_2 - l_\infty$ performance under dropouts. Figure 8 depicts the systems and filter mode. Figure 9 displays $z(k)$ (red dotted line) and $\mathbb{E}\{\hat{z}(k)\}$ (blue line) with dynamic quantization where the pink area represents the location of all 100,000 estimated signals. Figure 10 shows $\mathbb{E}\{e_z(k)\}$ where the pink area represents the location of all 100,000 error responses. The release interval is shown in Figure 11 and the quantization levels are displayed in Figure 12. Differing from the static quantization

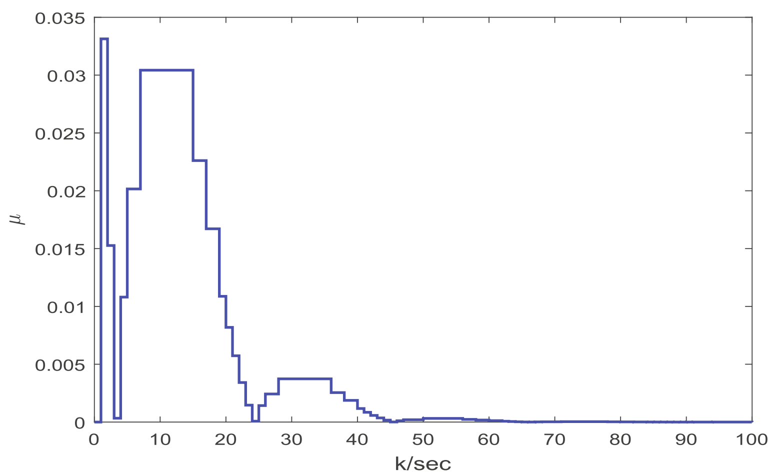

FIGURE 7 Quantization levels of H_∞ performance

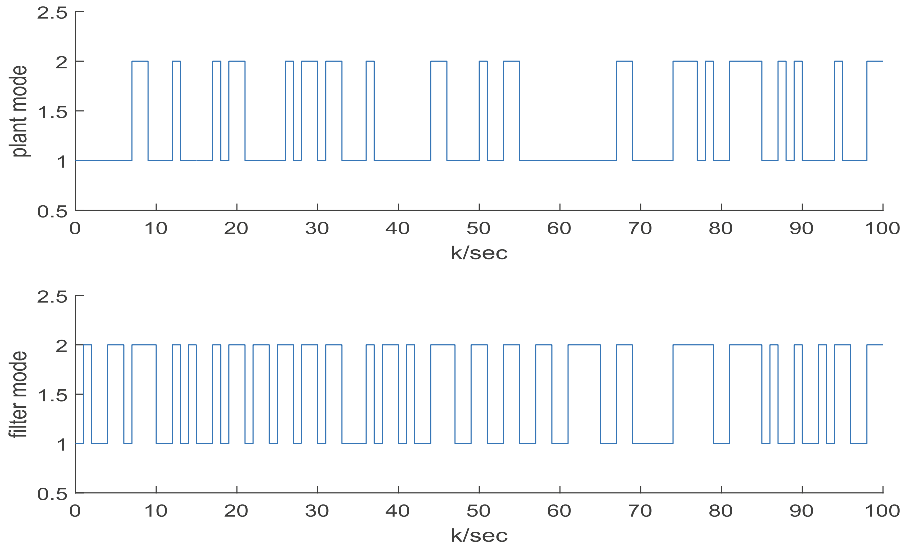

FIGURE 8 The evolution of systems and filter mode of $l_2 - l_\infty$ performance

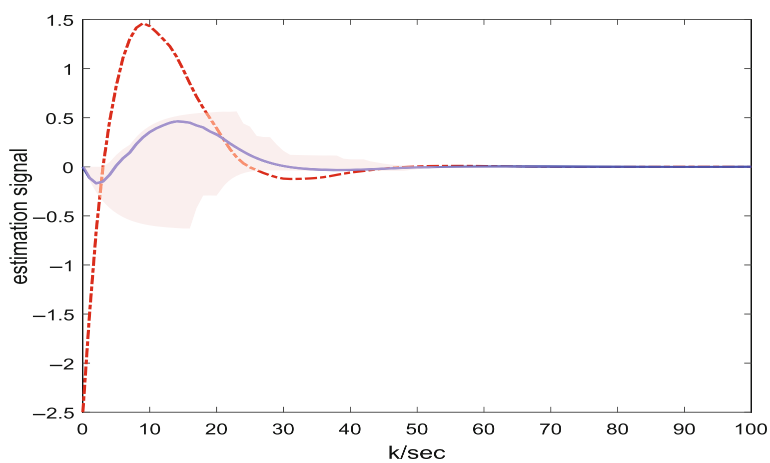

FIGURE 9 The estimation signal of systems and filter of $l_2 - l_\infty$ performance

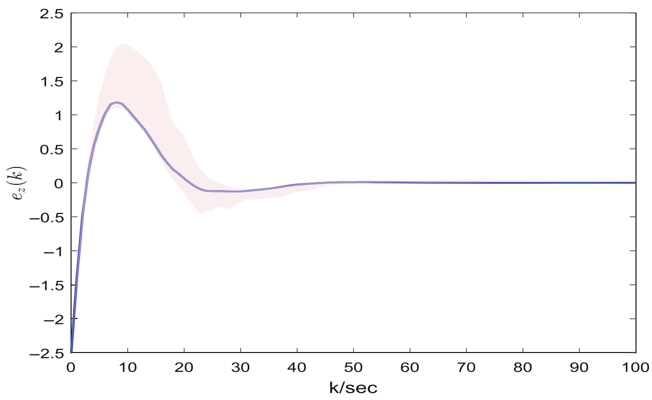

FIGURE 10 Error response of $l_2 - l_\infty$ performance

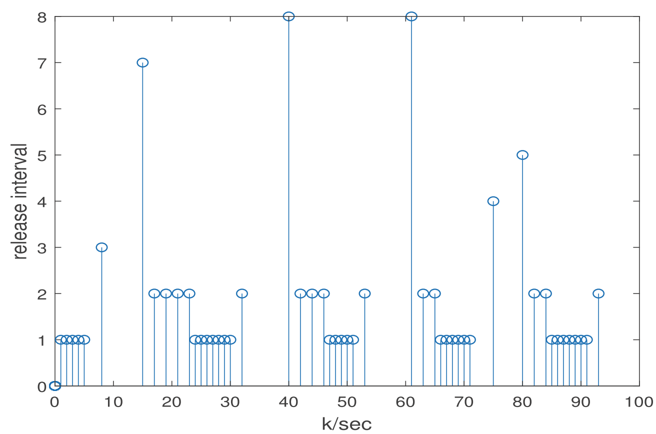

FIGURE 11 One sample of release interval and instants of $l_2 - l_\infty$ performance

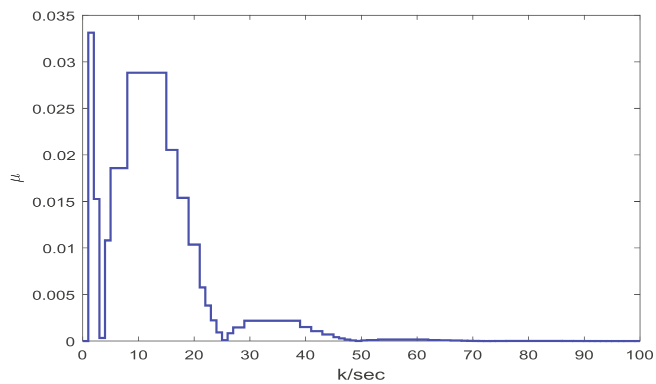

FIGURE 12 Quantization levels of $l_2 - l_\infty$ performance

levels, the quantization levels adopted in this paper change adaptively based on the signal to be quantized for mitigating performance degradation.

From the case I and case II, we can summarize that the extended dissipative performance of the networked FN-MJSs can be guaranteed by the event-triggered asynchronous filter with dynamic quantization.

5 | CONCLUSION

In this paper, the event-triggered asynchronous filtering for the networked FN-MJSs with dynamic quantization is studied. Within the general framework of extended dissipativity, an asynchronous filter is designed where the event-triggered scheme and the dynamic quantization technology is adopted to alleviate packet dropout. Based on the fuzzy-rule-independent Lyapunov function, sufficient conditions are given such that the error systems under quantization levels condition are stochastically stable with desired extended dissipative performance. Furthermore, the asynchronous filter is designed which is given in the form of linear matrix inequalities where the free-connection weighting matrices are utilized to deal with the time-varying MTPMs. Simulations are presented to examine the effectiveness of the asynchronous filter design where classical H_∞ performance and $l_2 - l_\infty$ performance are investigated.

ACKNOWLEDGMENTS

This work is supported by the National Key Research and Development Project under Grant 2018AAA0100802, the Anhui Provincial Natural Science Foundation under Grant 2008085MF198, and Institute of Artificial Intelligence under Grant 21KT010.

ORCID

Jin Zhu https://orcid.org/0000-0002-6038-4339

REFERENCES

1. Costa OLV, Fragoso MD, Marques RP. *Discrete-Time Markov Jump Linear Systems*. *Springer Science and Business Media*; 2006.
2. Zhu J, Wang LP, Spiryagin M. Control and decision strategy for a class of Markovian jump systems in failure prone manufacturing process. *IET Control Theory Appl.* 2012;6(12):1803-1811.
3. Assawinchaichote W, Nguang SK. H_∞ for fuzzy singularly perturbed systems with pole placement constraints: an LMI approach. *IEEE Trans. Signal Process*. 2004;52(6):1659-1667.
4. Ebrahimi Z, Asemani MH, Safavi AA. Observer-based controller design for uncertain disturbed Takagi-Sugeno fuzzy systems: A fuzzy wavelet neural network approach. *Int J Adapt Control Signal Process.* 2020;35(1):122-144.
5. Niu YJ, Dong W, Ji YD. Mode-independent H_∞ filtering for discrete-time Markov jump linear system with parametric uncertainties and quantized measurements. *Int J Adapt Control Signal Process.* 2016;30(7):957-971.
6. Mesquita A. Parsimonious Bayesian filtering in Markov jump systems with applications to networked control. *IEEE Trans. Automat Contr.* 2020;66(1):76-88.
7. Geromel JC, Deaecto GS, Colaneri P. Minimax control of Markov jump linear systems. *Int J Adapt Control Signal Process.* 2016;30(8-10):1152-1162.
8. Zhu J, Xia K, Ling Q, Chen W, Dullerud GE. Stabilization and optimization of discrete-time Markovian jump linear systems via mode feedback control. *IEEE Trans. Automat Contr.* 2022;67(12):6505-6520.
9. Wu ZG, Shi P, Su H, Chu J. Asynchronous $l_2 - l_\infty$ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities. *Automatica.* 2014;50(1):180-186.
10. Ning Z, Zhang L, Mesbah A, Colaneri P. Stability analysis and stabilization of discrete-time non-homogeneous semi-Markov jump linear systems: a polytopic approach. *Automatica.* 2020;120:109080.
11. Shen Y, Wu ZG, Shi P, Ahn CK. Model reduction of Markovian jump systems with uncertain probabilities. *IEEE Trans. Automat Contr.* 2020;65(1):382-388.
12. Li F, Shi P, Lim CC, Wu L. Fault detection filtering for non-homogeneous Markovian jump systems via a fuzzy approach. *IEEE Trans. Fuzzy Syst.* 2016;26(1):131-141.
13. Tian Y, Wang Z. Finite-time extended dissipative filtering for singular T-S fuzzy systems with non-homogeneous Markov jumps. *IEEE Trans. Cybern*. 2022;52(6):4574-4584. doi:10.1109/TCYB.2020.3030503
14. Zhang L, Cai B, Tan T, Shi Y. Stabilization of non-homogeneous hidden semi-Markov jump systems with limited sojourn-time information. *Automatica.* 2020;117:108963.
15. Kim SH. Asynchronous dissipative filter design of non-homogeneous Markovian jump fuzzy systems via relaxation of triple-parameterized matrix inequalities. *Inf Sci.* 2019;478:564-579.
16. Zhu J, Li CX, Dullerud GE. Asynchronous H_∞ control for two-dimensional hidden Markovian jump systems with partly known mode observation conditional probabilities. *Int J Robust Nonlinear Control*. 2020;30(8):3344-3364.
17. Hua M, Zheng D, Deng F, Fei JT, Cheng P. H_∞ filtering for non-homogeneous Markovian jump repeated scalar nonlinear systems with multiplicative noises and partially mode-dependent characterization. *IEEE Trans. Syst Man Cybern Syst*. 2021;51(5):3180-3192.
18. Yin Y, Shi P, Liu F, Teo KL, Lim CC. Robust filtering for nonlinear non-homogeneous Markov jump systems by fuzzy approximation approach. *IEEE Trans. Cybern.* 2014;45(9):1706-1716.
19. Liu J, Ran G, Huang Y, Han C, Yu Y, Sun C. Adaptive event-triggered finite-time dissipative filtering for interval type-2 fuzzy Markov jump systems with asynchronous modes. *IEEE Trans. Cybern.* 2022;52(9):9709-9721.
20. Shen M, Ye D, Wang QG. Event-triggered H_∞ filtering of Markov jump systems with general transition probabilities. *Inf Sci.* 2017;418:635-651.
21. Yan H, Sun J, Zhang H, Zhan X, Yang F. Event-triggered H_∞ state estimation of 2-DOF quarter-car suspension systems with non-homogeneous Markov switching. *IEEE Trans. Syst Man Cybern Syst.* 2018;50(9):3320-3329.

22. Tao J, Lu R, Su H, Shi P, Wu ZG. Asynchronous filtering of nonlinear Markov jump systems with randomly occurred quantization via T-S fuzzy models. *IEEE Trans. Fuzzy Syst.* 2017;26(4):1866-1877.
23. Zhao D, Liu Y, Liu M, Yu J, Shi Y. Network-based robust filtering for Markovian jump systems with incomplete transition probabilities. *Signal Process.* 2018;150:90-101.
24. Hua M, Qian Y, Deng F, Fei J. Filtering for discrete-time Takagi-Sugeno fuzzy non-homogeneous Markov jump systems with quantization effects. *IEEE Trans. Cybern.* 2022;52(2):982-995.
25. Liberzon D. Hybrid feedback stabilization of systems with quantized signals. *Automatica.* 2003;39(9):1543-1554.
26. Lam HK, Wu L, Zhao Y. Linear matrix inequalities-based membership function-dependent stability analysis for non-parallel distributed compensation fuzzy-model-based control systems. *IET Control Theory Appl.* 2014;8(8):614-625.
27. Kim HJ, Park JB, Joo YH. Decentralised H_∞ fuzzy filter for non-linear large-scale systems under imperfect premise matching. *IET Control Theory Appl.* 2015;9(18):2704-2714.
28. Costa OL, Fragoso MD, Todorov MG. A detector-based approach for the H_2 control of Markov jump linear systems with partial information. *IEEE Trans. Automat Contr.* 2015;60(5):1219-1234.
29. Zhang B, Zheng WX, Xu S. Filtering of Markovian jump delay systems based on a new performance index. *IEEE Trans. Circ Syst I Regul Pap.* 2013;60(5):1250-1263.
30. Zhang X, Wang H, Stojanovic V, et al. Asynchronous fault detection for interval type-2 fuzzy non-homogeneous higher-level Markov jump systems with uncertain transition probabilities. *IEEE Trans. Fuzzy Syst*. 2022;30(7):2487-2499. doi:10.1109/TFUZZ.2021.3086224
31. Horn RA, Johnson CR. *Matrix Analysis*. 2nd ed. *Cambridge University Press*; 2013.
32. Dong S, Wu ZG, Shi P. Reliable filter design of fuzzy switched systems with imprecise modes. *Control and Filtering of Fuzzy Systems with Switched Parameters*. *Springer*; 2020.
33. Shi P, Su XJ, Li FB. Dissipativity-based filtering for fuzzy switched systems with stochastic perturbation. *IEEE Trans. Automat Contr.* 2016;61(6):1694-1699.
34. Ding DW, Yang GH. Fuzzy filter design for non-linear systems in finite-frequency domain. *IEEE Trans. Fuzzy Syst.* 2010;18(5):935-945.
35. Liu H, Ho DW, Sun F. Design of H_∞ filter for Markov jumping linear systems with non-accessible mode information. *Automatica.* 2008;44(10):2655-2660.
36. Liu K, Fridman E, Johansson KH. Dynamic quantization of uncertain linear networked control systems. *Automatica.* 2015;59:248-255.

How to cite this article: Zhu J, Xie Z, Zhao Y-B, Dullerud GE. Event-triggered asynchronous filtering for networked fuzzy non-homogeneous Markov jump systems with dynamic quantization. *Int J Adapt Control Signal Process*. 2023;37(3):811-835. doi: 10.1002/acs.3551